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Simplified Engineering
for
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BY

HARRY PARKER, M.S.

Professor of Architectural Construction
University of Pennsylvania

SECOND EDITION

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PREFACE TO SECOND EDITION

In writing *Simplified Engineering for Architects and Builders* the author had in mind a very definite purpose, namely, ". . . to present to those having little or no knowledge of the subject simple solutions of everyday structural problems." It is gratifying to know that this book has been of practical value and has justified its preparation.

It is surprising, however, to realize how quickly a book of this character becomes out-of-date. New building code requirements with corresponding changes in unit stresses, even in a short space of time, are much more numerous than would be expected. Thus, in order to retain its worth, it has been necessary to revise or supplement nearly all the pages of this book.

In addition to the introduction of new unit stresses and solutions of examples, new tables have been added, the former problems revised where necessary, and an almost equal number of new problems presented.

The author has endeavored to confine discussions and solutions of problems to illustrations of basic principles by avoiding complicated mathematics and involved theories and presenting illustrative examples that have a practical value to those engaged in practice.

In its new form, it is hoped that *Simplified Engineering for Architects and Builders* will continue in its usefulness and be of even greater value to those who seek knowledge concerning the design of structural members that occur so frequently in building construction.

HARRY PARKER

High Hollow
Southampton, Bucks Co., Pa.
September, 1946

PREFACE TO FIRST EDITION

To the average young architectural draftsman or builder, the problem of selecting the proper structural member for given conditions appears to be a very difficult task. Most of the numerous books on engineering which are available assume that the reader has previously acquired a knowledge of fundamental principles, and thus are almost useless to the beginner. It is true that some engineering problems are exceedingly difficult, but it is also true that many of the problems that occur so frequently are surprisingly

simple in their solution. With this in mind, and with a consciousness of the seeming difficulties in mastering structural problems, this book has been written.

In order to understand the discussions of engineering problems, it is essential that the student have a thorough knowledge of the various terms which are employed. In addition, basic principles of forces in equilibrium must be understood. The first section of this book, "Principles of Mechanics," is presented for those who have not had such technical preparation, as well as for those who wish a brief review of the subject. Following this section are structural problems involving the most commonly used building materials, wood, steel, reinforced concrete and roof-trusses. A major portion of the book is devoted to numerous problems and their solution, the purpose of which is to explain practical procedure in the design of structural members. Similar examples are given to be solved by the student. Although handbooks published by the manufacturers are necessities to the more advanced student, a great number of appropriate tables are presented herewith so that sufficient data are directly at hand to those using this book.

Care has been taken to avoid the use of advanced mathematics, a knowledge of arithmetic and high-school algebra being all that is required to follow the discussions presented. The usual formulas employed in the solution of structural problems are given with explanations of the terms involved and their application, but only the most elementary of these formulas are derived. These derivations are given to show how simple they are and how the underlying principle involved is used in building up a formula that has a practical application.

No attempt has been made to introduce new methods of calculation, nor have all the various methods been included. It has been the desire of the author to present to those having little or no knowledge of the subject simple solutions of everyday structural problems. Whereas thorough technical training is to be desired, it is hoped that this presentation of fundamentals will provide valuable working knowledge and, perchance, open the doors to more advanced study.

HARRY PARKER

Philadelphia, Pa.
March, 1938

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SUGGESTIONS

Those unfamiliar with the terms and basic principles of mechanics should not fail to study Section I before attempting the succeeding sections. A determination to master the subject and a willingness to work diligently are all that are necessary for success. The following suggestions will prove helpful.

First, take up each item in the sequence presented, and be certain that each is thoroughly understood before continuing with the next.

Second, each problem is prepared to illustrate some basic principle or theory. Make no attempt at solving a problem until you have read carefully and fully understand exactly what is wanted. In each problem certain quantities are known; the unknown quantity to be found is the solution of the problem. Write down the known quantities or data and consider the principle or formula which governs their relation. This is the key to the solution.

Third, whenever possible, make a sketch noting the data given. Such diagrams frequently show at a glance the problem to be solved and the procedure necessary for its solution.

Fourth, form the habit of noting the denominations or names of the quantities, as, for instance, 19.2in^2 or 30,000″#. Failure to do this often results in errors which otherwise would be avoided.

SECTION I

PRINCIPLES OF MECHANICS

DIRECT STRESSES

1. **Forces.** A force may be defined as that which exerts motion, pressure, or tension. In building construction we are concerned only with forces in equilibrium or at rest. If a force is at rest it must be held so by some other force or forces. *Statics* pertains to bodies at rest under the action of forces. A steel column supports a given load which, owing to gravity, is downward. The column transfers the load to the footing below; the resultant pressure on the footing equals the load in magnitude, its direction is upward and may be called the reaction. The two forces are opposite in direction, have the same line of action, and are equal in magnitude. The result is equilibrium, that is, no motion. The units of force are pounds, tons, kilograms, and so on. In engineering practice the word *kip* is frequently used; it merely means a thousand pounds. Thus 30 kips might also be written 30,000 pounds.

2. **Direct Stresses.** Suppose that the load on a column is 100,000 pounds and that the cross-section area of the column is 10 square inches.[1] Then each square inch of steel resists 100,000 divided by 10 or 10,000 pounds. In this instance, the unit stress in the column is 10,000 pounds per square inch. A *stress* in a body is an internal resistance to an external force. The total stress in the column is 100,000 pounds while the stress per unit of area is 10,000 pounds per square inch.[2] The three different kinds of stresses which concern us are *tension, compression,* and *shear.* The stress which accompanies the tendency of a force to twist a bar is called *torque,* but this does not concern us in building construction. Unless otherwise noted, we shall assume that forces are axial and that the

[1] 10 square inches may also be written $10in^2$.
[2] 10,000 pounds per square inch may be written $10,000\#/in^2$.

stresses are evenly distributed over the cross-section area of the body under stress.

Let us call the load or external force P, the area of the cross section A and the unit stress f. It is obvious that the load divided by the section area will give the unit stress. This may be stated

$$f = \frac{P}{A} \quad \text{or} \quad P = Af \quad \text{or} \quad A = \frac{P}{f}$$

This is a fundamental principle. Remember, when using it, it is assumed that the stresses are equally distributed. Note also that if any two of the quantities are known the third or unknown may be found.

3. **Deformation.** Whenever a force acts upon a body there is an accompanying change in shape or size of the body. This is called deformation. Regardless of the magnitude of the force, the deformation is always present, although often it is so small that it

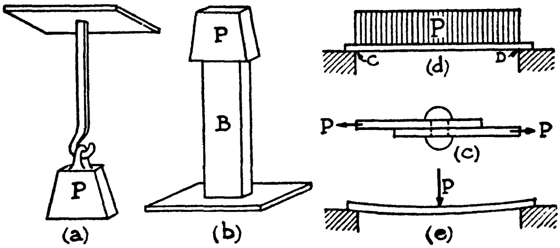


FIG. 1

is difficult to measure even with the most accurate instruments. In designing structures it is often necessary that we know what the deformation in certain members will be. A floor joist, for instance, may be sufficiently large to support safely a given load but it may deflect (the deformation that occurs with bending) to such an extent that the floor shakes or the plastered ceiling below may crack. For the usual cases we can readily determine what the deformation will be. This will be explained later.

4. **Tension.** When a force acts upon a body in such a manner that the body tends to lengthen or pull apart, the force is called tensile. Figure 1 (a) shows a rod supported from above with a load

P suspended at the bottom. The force *P* tends to elongate the bar and consequently is a tensile force. The stresses in the bar due to the tensile force *P* are called tensile stresses.

EXAMPLE

EXAMPLE. A wrought iron rod having a diameter of 1½″ is used in a roof truss. If the tensile force supported is 20,000#, what is the unit stress?

SOLUTION. To find the area of the rod we square the radius and multiply by 3.1416. Therefore the area equals $0.75^2 \times 3.1416 = 1.76\text{in}^2$. The load is 20,000#. These are all the data; what we seek is the unit stress. The formula giving the relation between these three quantities is $f = \frac{P}{A}$, Art. 2. Hence $f = \frac{20,000}{1.76} = 11,363\#/\text{in}^2$, the unit stress in the rod. Be sure to note that the unit stress is in terms of pounds per square inch because *P* is in units of pounds and *A* is in square inches.

5. **Compression.** Figure 1 (*b*) indicates the force *P* resting on the member *B*. When the force acting upon a body has a tendency to shorten it, the force is called compressive and the stresses within the member are compressive stresses. A typical example of compression is a column having a load on its upper end.

EXAMPLE

EXAMPLE. Suppose we have a short timber post of No. 1 Structural grade of longleaf Southern pine, 8″ x 8″ in cross-section area, and wish to know what load it will safely support.

SOLUTION. The nominal size of the timber is 8″ x 8″, but the "American standard dressed size," or actual size, is 7½″ x 7½″; therefore the area of the cross section is 56.25in^2; see Table 12. By consulting Table 3 we find that the allowable unit stress for this grade of timber is $1,000\#/\text{in}^2$. The key to the solution of this problem is again the fundamental formula, $P = fA$. Therefore, when the known values are substituted, $P = 1,000 \times 56.25$ or $P = 56,250\#$, the safe load.

6. **Shear.** A shearing stress occurs when we have two forces acting in opposite directions not in the same line. Forces acting

as a pair of scissors, tending to cut a body, is one illustration. Figure 1 (*c*) illustrates two plates held together by a rivet. The forces P acting on the plates tend to shear the rivet on an area equal to the area of the rivet at the plane between the two plates. The load P, Fig. 1 (*d*), rests upon a beam which in turn is supported on walls at its ends. It is obvious that there is a tendency for the beam to fail by shearing at the points C and D.

EXAMPLE

EXAMPLE. The force P in the plates illustrated in Fig. 1 (*c*) is 5,000# and the rivet has a diameter of ¾″. What is the shearing unit stress?

SOLUTION. A ¾″ rivet has a cross-section area of 0.4418in^2; see Table 25-A. The fundamental formula is $f = \frac{P}{A}$. Substituting, $f = \frac{5,000}{0.4418} = 11,317\#/\text{in}^2$, the shearing unit stress.

7. **Bending.** Figure 1 (*e*) illustrates a simple beam with a concentrated load P at the center of the span. This is an example of bending or flexure. The fibers in the upper part of the beam are in compression, and those in the lower part are in tension. These stresses are not equally distributed over the cross section and are discussed in Arts. 38 and 39.

PROBLEMS

7-A. A wrought iron bar receives a tensile force of 40,000#. If the allowable unit tensile stress of wrought iron is 12,000#/in^2, what should be the area of the bar?

7-B. What load may be placed on a short timber post whose actual cross-section dimensions are 9½″ x 9½″ if the allowable compressive unit stress is 1,200#/in^2?

7-C. A 4″ x 4″ x ½″ angle has a cross-section area of 3.75in^2. If the angle be subjected to a tensile force of 60,000#, what is the tensile unit stress?

7-D. Two rivets are used at the end of an angle of a truss, the total stress in the member being 16,240#. What should be their diameters if the shearing unit stress is 13,500#/in^2?

7-E. The allowable bearing power of a foundation bed is 4 tons per square foot. What should be the length of a side of a square footing if the load on the column and footing is 240,000#?

7-F. A steel bolt in a truss is to resist a shearing force of 16,470#. What should be its diameter if the allowable shearing unit stress is 13,500#/in^2?

7-G. A short length of a 12″ steel I-beam weighing 31.8 pounds per linear foot is to be used as a post. What load will it support if the compressive unit stress is 15,000#/in^2? (For area of I-beam see Table 5.)

7-H. A short, hollow, cast iron column is circular in cross section, the outside diameter being 10″ and the thickness of the shell ¾″. If the allowable unit stress is 14,000#/in^2, what load will the column support?

7-I. Determine the minimum cross-section area of a steel bar required to support a tensile stress of 50,000#. (See Table 2.)

7-J. Three 1″ diameter rivets are used to connect two steel plates. If the allowable shearing stress of rivet steel is 13,500#/in^2, what load will the connection support with respect to shear?

7-K. A short square timber post supports a load of 112,400#. If the allowable compressive unit stress is 1,000#/in^2, timber of what stock size should be used? (See Table 12.)

STRESSES WITH NEW NAMES

8. **Hooke's Law.** In the seventeenth century there lived in England a gentleman named Robert Hooke, a mathematician and physicist. As a result of experiments with clock springs, he developed the principle that "deformations are directly proportional to stresses." In other words, if a force produces a certain deformation, twice the force will produce twice the amount of deformation. This law of physics is of the utmost importance, though unfortunately good Mr. Hooke did not carry his experiments quite far enough, for it may readily be shown that Hooke's Law holds true only up to a certain limit.

9. **Elastic Limit.** Suppose that we place a bar of structural steel having a section area of 1in^2 in a machine made for performing tensile tests. We measure its length accurately and then apply a tensile force of 5,000#. We measure the length again and find that the bar has lengthened a definite amount which we will call X inches. On applying 5,000# more, we note that the amount of lengthening is $2 \times X$, or twice the amount noted after the first 5,000#. If the test is continued, we find that, for each 5,000# additional, the length of the bar increases the same amount that was noted when the first unit of 5,000# was applied. That is, the deformations are directly proportional to the stresses. Thus far Hooke's Law has held true. But, and this is the significant point, after we have applied about 35,000# the length increases more

than X inches for each additional 5,000#. This unit stress, about 35,000#/in^2, which varies with different specimens, is called the elastic limit. It may be defined as the unit stress beyond which the deformations increase in a faster ratio than the applied loads. The elastic limits of cast iron and wrought iron are given in Table 2.

Here is another result that is noted. If we make the test again, we discover that, if any applied load, less than the elastic limit, is removed, the bar returns to its original length. If the unit stress is greater than the elastic limit, and the force is removed, we find that the bar has permanently increased its length. This phenommenon permits another way of defining the elastic limit: It is that unit stress beyond which the material does not return to its original length when the load is removed.

Another term used in connection with these tests is the *yield point.* It is found during tests that some materials, steel and wrought iron for instance, show increases in deformation without any increase in stress; the unit stress at which this deformation occurs is called the yield point. The yield point, although slightly higher than the elastic limit, is almost identical to the elastic limit. Non-ductile materials such as wood and cast iron have poorly defined elastic limits and no yield point.

10. **Ultimate Strength.** If a load of sufficient magnitude is applied to a test specimen, rupture occurs. The unit stress within the bar just prior to its breaking is called the ultimate strength.

For the steel bar referred to in the preceding article, failure occurs at a unit stress of about 60,000#/in^2. A structural member stressed beyond the elastic limit is usually unsafe even though the ultimate strength of the material is not reached. The ultimate strength for various building materials is given in Table 2. Note that these average values are given for tension, compression, and shear, and the units are pounds per square inch.

11. **Factor of Safety.** The factor of safety is a term often used in connection with engineering. It is important to know exactly what it means. Imagine that a wrought iron rod in a roof truss has an actual tensile stress of 9,600#/in^2, and that the ultimate tensile strength of the material is 48,000#/in^2. The ratio of the ultimate strength of a material to the unit stress is the factor of safety. In this instance, it is 5, which is the result of dividing

48,000 by 9,600. In other words, the factor of safety is the number which results from dividing the ultimate strength of a material by the unit stress.

Error in understanding this term may cause failures from overloading members. The wrought iron rod referred to above will not carry a load 5 times the allowable unit stress, because failure may result if the unit stress exceeds the elastic limit, in this case 25,000#/in^2. Consequently, the term *factor of safety* is not employed extensively today.

If one should be required to pass judgment upon the safety of a given structure, the problem resolves itself into considering each

Table 1. Factors of Safety

Material	Factor of Safety when Used in Buildings
Brick and stone. .	15
Timber	8–10
Cast iron....... .	6
Wrought iron......	4
Structural steel.....	4

structural element, finding its actual unit stress under the existing loads, and comparing this stress with the ultimate strength or with the allowable stress prescribed by the building regulations. Finding the factor of safety is called investigation.

Owing to thė difference in nature of various materials, it is advisable to require higher factors of safety for some materials than for others. Table 1 is given as a guide; the working unit stresses derived by this method are approximate, and the stresses permitted by building regulations should always be consulted in actual design.

12. **Working Unit Stress.** The working unit stress or allowable unit stress is the stress used in designing structural members. It is the unit stress selected or specified as the maximum permissible

Table 2. Average Physical Properties of Various Building Materials

Material	Elastic Limit, Pounds per Square Inch		Ultimate Strength, Pounds per Square Inch			Allowable Working Unit Stress, Pounds per Square Inch				Modulus of Elasticity, Pounds per Square Inch	Weight, Pounds per Cubic Foot
	Tension	Compression	Tension	Compression	Shear	Tension	Compression	Shear	Bending		
Concrete (1:2:4) *				2,000			500	40		2.000,000	150
Brick masonry				2.000			100–250				120
Stone masonry				2 500			200–400				145
Cast iron	12,000	25,000	25,000	80 000	20,000	3,000	14,000	4,000	8,000	12,000,000	450
Wrought iron	25,000	25,000	48.000	48,000	40,000	12,000	12,000	8,000	12,000	27,000,000	485
Steel, structural	35.000	35,000	60,000	60.000	45.000	18.000	18,000	12,000 †	18.000	30.000,000	490
Timber,‡ parallel to grain	3,000	3,000	10,000	8,000	500	1,000	1,000	80 (horizontal)	1,000	1,200,000	40
perpendicular to grain					3,000		300	400			

* For allowable stresses in concrete see Table 30.
† Allowable shearing unit stress for rivets is 13,500#/in².
‡ For allowable stresses in timber see Table 3.

in a member. From the previous discussion, we can see that working unit stresses should be well within the elastic limit, and occasionally we find in a building code the statement that the working unit stress for a given material must be ½ or ⅓ the elastic limit. Again, the specification may give the exact unit stress to be used with no reference to the factor of safety or elastic limit.

Table 2 is presented as a ready reference for those using this book. The values given are approximate. Should it be desired to design structural members for actual use, the building code or laws governing the construction of buildings in the particular locality should be consulted for specific requirements. More complete information concerning concrete will be found in Section IV. The strength of timber depends primarily on its species and grade, and a condensed table (Table 3) lists some of the most commonly used timbers and their allowable working stresses.

The more recent building codes have increased the allowable stresses for structural steel. The allowable tensile stress and the allowable extreme fiber stress are now 20,000#/in^2 instead of 18,000#/in^2 as given in Table 2. The allowable shearing stress in the webs of beams and plate girders is 13,000#/in^2. Unless otherwise noted, the stresses given in Table 2 will be used in the following examples and problems.

13. **Design.** The stresses we have discussed thus far have been direct or axial stresses. This means that the stresses are assumed to be equally distributed over the cross section. By the use of the formula $P = Af$ and the accompanying tables, we can now solve three types of problems: first, the design of structural members; second, determination of safe loads; third, investigation of members for safety.

See how simple it is;

EXAMPLES

Example. Let it be required to design (determine the size) of a short square post of Douglas fir, select structural grade, having an axial load of 32,500#.

Solution. By referring to Table 3, we find the allowable compressive unit stress for this timber to be 1,200#/in^2. Substituting

Table 3. Allowable Working Stresses for Various Structural Woods

Species	Grade	Allowable Unit Stresses in Pounds per Square Inch: Extreme fiber in bending and tension parallel to grain	Horizontal shear	Compression: Perpendicular to grain	Compression: Parallel to grain	Modulus of Elasticity in Pounds per Square Inch
Southern cypress	1,400# *f* Southern cypress	1,400	120	300		1,200,000
	1,100# *f* Southern cypress	1,100	100	300		1,200,000
	1,200# *c* Southern cypress				1,200	
	1,000# *c* Southern cypress				1,100	
Douglas fir (Coast region)	Dense Select Structural	1,800	120	380	1,300	1,600,000
	Select Structural	1,600	100	345	1,200	1,600,000
	1,200# *f* framing and joist	1,200	100	325		1,600,000
	900# *f* framing and joist	900	100	325		1,600,000
Eastern hemlock	Select Structural	1,100	70	300	700	1,100,000
	1,000 SG Eastern hemlock	1,000	52	300		1,100,000
	900 SG Eastern hemlock	900	52	300		1,100,000
	800 SG Eastern hemlock	800	52	300		1,100,000
Longleaf Southern pine	Select Structural	2,000	100	380	1,450	1,600,000
	Prime Structural	1,800	100	380	1,300	1,600,000
	Merchantable Structural	1,600	100	380	1,200	1,600,000
	Structural Square Edge and Sound	1,600	100	380	1,200	1,600,000
	No. 1 Structural	1,400	100	380	1,000	1,600,000
Shortleaf Southern pine	Dense Select Structural	2,000	100	380	1,450	1,600,000
	Dense Structural	1,800	100	380	1,300	1,600,000
	Dense Structural Square Edge and Sound	1,600	100	380	1,200	1,600,000
	Dense No. 1 Structural	1,400	100	380	1,000	1,600,000
Redwood	1,600# *f* Close-grained	1,600	80	267		1,200,000
	Dense Select All-Heart	1,400	80	267		1,200,000
	Select All-Heart	1,200	70	267		1,200,000
Eastern spruce	1,200# *f* Eastern spruce	1,200	90	250		1,200,000
	1,100# *f* Eastern spruce	1,100	80	250		1,200,000
	1,000# *f* Eastern spruce	1,000	80	250		1,200,000

the known values in the formula $A = \frac{P}{f}$, $A = \frac{32{,}500}{1{,}200}$ or $A = 27\text{in}^2$, the number of square inches required in the post. A 6″ x 6″ timber has actual dimensions of 5½″ x 5½″ or 30.25in^2 (Table 12), and this is the size we will select.

EXAMPLE. A running track in a gymnasium is hung from trusses by wrought iron rods. Each rod supports a floor area of 56ft^2. Rods of what size should be used if the assumed live and dead floor load is $200\#/\text{ft}^2$?

SOLUTION. The total tensile load on each rod will be 200 x 56 or 11,200#. Table 2 gives the allowable tensile unit stress of wrought iron to be $12{,}000\#/\text{in}^2$. Therefore, $A = \frac{P}{f}$ or $A = \frac{11{,}200}{12{,}000}$ $= 0.93\text{in}^2$, the required area of the rod. A rod having a diameter of 1⅛″ has a cross section of 0.99in^2, and this may be used if the ends are upset. The upset is necessary because the cross-section area of a 1⅛″ rod at the root of the thread is only 0.694in^2; the books call this the net area. The net area of a 1⅜″ rod is 1.057in^2, which may be used without upsets.

PROBLEMS

13-A. What should be the area of a steel rod to support a tensile load of 24,000# if the unit stress is ⅓ the elastic limit?

13-B. What should be the diameter of a short bar of cast iron to support a compressive load of 98,800#?

13-C. Two steel bars are riveted together with ¾″ rivets. The total stress on the bars is 42,000#. If only shear is considered, how many rivets should be used?

13-D. The bars in the previous problem are ½″ thick. If the hole required for ¾″ rivets is ⅞″ in diameter, what should be the width of the bars to resist the tension of 42,000#?

13-E. What should be the diameter of a column footing, circular in plan, if the load on the column is 75,000# and the allowable bearing strength of the foundation bed is 4 tons per square foot?

13-F. A steel beam resting on two supports at its ends has a total uniformly distributed load of 120,000#. What should be the area of the bearing plates if they rest upon stone masonry?

13-G. A short, square, longleaf Southern yellow pine, No. 1 Structural grade post supports a load of 56,000#. What should be its dimensions?

13-H. A wooden lintel supporting a uniformly distributed load of 150,000# is supported at each end by timber posts. What should be the minimum cross-section area of each post if the allowable compressive stress is 1,200#/in^2?

13-I. The force exerted by a steel beam on a wall laid up with common brick and cement mortar is 60,000#. What should be the minimum area of the bearing plate? (See Table 24.)

14. **Safe Loads.** For structural members having direct stresses, the problem of computing the safe loads consists merely in solving for P in the formula $P = Af$, the size of the member being given and a safe working stress being assumed.

EXAMPLES

EXAMPLE. A 6″ x 8″ timber having an allowable tensile stress of 1,200#/in^2 is used as the bottom chord of a roof truss. What safe load will it support?

SOLUTION. From its position in the truss, this member is in tension. Since it is necessary to cut out for notches and bolts, only 60 per cent of the gross area will be considered as effective. A 6″ x 8″ timber has actual dimensions of 5½″ x 7½″ (Table 12), or 41.25in^2; 60 per cent of 41.25 = 24.75in^2; therefore assume the net area to be 25in^2. Since the allowable tensile unit stress of this particular timber is 1,200#/in^2, and $P = Af$, $P = 25 \times 1{,}200 =$ 30,000#, the safe load.

EXAMPLE. What safe load may be placed on a 4′ 6″ x 4′ 6″ concrete column footing if the bearing power of the foundation bed is 5 tons per square foot?

SOLUTION. $4.5 \times 4.5 = 20.25$ft^2, the area of the footing; 5 tons $= 5 \times 2{,}000 = 10{,}000$#. Then $P = Af$ or $P = 20.25 \times 10{,}000 =$ 202,500#, the safe load.

PROBLEMS

14-A. A steel rod supporting a chandelier has a diameter of 1¼″. What safe load will it support?

14-B. A short, hollow, cast iron column has an outside diameter of 10″, and the thickness of the shell is 1″. What safe load will it support?

14-C. What load may be placed on a short pier built of common brick with cement mortar, 13″ x 17″ in cross section? (See Table 24.)

14-D. What safe tensile load may be applied to a 1″ steel rod having upset screw ends?

14-E. What safe load will a short 12″ x 12″ longleaf Southern pine post support, if the grade is No. 1 Structural?

14-F. If a wrought iron bolt in tension has a diameter of 1½″ and a head 1″ long, what safe load will the bolt head resist with respect to shear?

14-G. What safe load may be placed on a stone-concrete pier 18″ x 20″ in cross section? (See Table 24.)

14-H. Two 4″ x 4″ x ¾″ steel angles are used as a tension member of a truss. What is the safe load on the member? (See Table 10.)

14-I. Two ⅞″ rivets are used to connect two steel bars. What is the safe load on the connection with respect to shear if the allowable shearing unit stress of rivet steel is 13,500#/in^2?

15. **Investigation.** To investigate a structural member, we simply find the actual unit stress and compare this with either the ultimate strength, the elastic limit, or the allowable unit stress required by building regulations. Of the three terms in the formula $f = \frac{P}{A}$, P and A are data and f remains to be found.

EXAMPLES

EXAMPLE. An angle in a roof truss resists a stress of 25,000#. At its end connection, it has four ¾″ diameter rivets. If only shear in the rivets is considered, is the joint safe?

SOLUTION. A ¾″ rivet has an area of 0.4418in^2 (Table 25-A). If the allowable shearing unit stress of rivets is assumed to be 13,500#/in^2, each rivet will safely resist 0.4418 × 13,500 or 5,964#. Four rivets will resist 4 × 5,964 or 23,856#. Since this is less than 25,000#, the load on the truss member, the joint is unsafe. Five rivets will be required.

EXAMPLE. A short 10″ x 10″ Eastern hemlock post, select Structural grade, has an axial load of 60,000#. Is the member safe?

SOLUTION. The actual dimensions of a 10″ x 10″ timber are 9½″ x 9½″ (Table 12), and the area is 90.25in^2. The unit stress in the member will be $f = \frac{P}{A}$ or $f = \frac{60,000}{90.25} = 664\#/\text{in}^2$. Since this is less than the allowable, 700#/in^2 given in Table 3, the member is safe.

EXAMPLE. A steel angle 4″ x 3″ x ⅜″ has a tensile load of 50,000#. What is the factor of safety? Is it large enough?

SOLUTION. An angle having the dimensions given has an area of 2.48in^2 (Table 11). The actual unit stress on the member will be $f = \frac{P}{A}$ or $f = \frac{50,000}{2.48} = 20,161\#/\text{in}^2$. If the ultimate tensile strength of steel is 60,000#/in^2 (Table 2), the factor of safety will be $\frac{60,000}{20,161} = 2.9$. This is not high enough (see Table 1). Most building regulations require that the unit stress of structural steel does not exceed 18,000#/in^2, or 20,000#/in^2.

PROBLEMS

15-A. A 10″ x 12″ bearing plate, under the end of a beam, rests on a brick wall laid up with cement mortar. The load on the plate is 42,000#. Is the plate large enough? What should be its width if the depth is 12″? (See Table 24.)

15-B. The shearing stress on each ⅞″ rivet of a lap joint is 3,005#. What is the factor of safety?

15-C. Two steel bars, each ½″ x 2″ in cross section, are held together by four ¾″ rivets. What tensile force applied to the bars would cause failure? How would failure occur? (Deduct a ⅞″ hole for a ¾″ rivet.)

15-D. A wrought iron rod 2″ in diameter is subjected to a tensile force of 50,250#. What is the factor of safety? Is it large enough?

15-E. A brick tower, laterally supported, is 200′ high. Compute the compressive unit stress on the brickwork at the base of the tower.

15-F. A short steel column has a cross-section area of 15.21in^2 and an axial load of 190 tons. What is the factor of safety? Is it large enough?

15-G. A tensile load of 167,300# is attached to the end of a structural steel bar whose diameter is 3″. What is the factor of safety? Is it large enough? If not, what should be the minimum area of the bar?

15-H. A stone pier 16″ x 20″ in cross section supports a load of 221,340#. Is the pier safe?

LENGTHENING AND SHORTENING OF BARS

16. **Deformation.** We have seen that if a bar is subjected to a force a deformation results. Also, if the unit stress in the bar does not exceed the elastic limit of the material, the deformations are directly proportionate to the stresses. And now we will see how very easy it is to compute the magnitude of these deformations. The key to this is to know a number, corresponding to the material with which we are dealing, which represents its stiffness. The name given to this term is rather formidable but actually it is nothing more than a ratio of one quantity to another.

17. Modulus of Elasticity. Suppose we place a steel rod 1in^2 in cross section in a testing machine and apply a tensile force of 2,000#. Its length becomes greater and, although we cannot detect it with the naked eye, the deformation can be measured by scientific instruments. If we put in the machine a piece of wood having the same dimensions as the steel rod, and apply the same force, we find the deformation to be greater, probably 30 times greater. We might say that the steel has a greater degree of stiffness than the wood. The number which represents the degree of stiffness of a material is known as the modulus of elasticity. We represent this quantity by the letter E and define it thus: The modulus of elasticity of a material is the unit stress divided by the unit deformation. Let us call the modulus of elasticity E, the unit stress f and the unit deformation s. Then, by definition,

$$E = \frac{f}{s}$$

From Art. 2, we remember that $f = \frac{P}{A}$. It is obvious that, if l represents the length of the member and e the total deformation, then s, the unit deformation, must equal the total deformation divided by the length or $s = \frac{e}{l}$. Now, if we substitute these values in the equation which was determined by definition,

$$E = \frac{f}{s} = \frac{\frac{P}{A}}{\frac{e}{l}} = \frac{P}{A} \times \frac{l}{e}$$

This can also be written

$$e = \frac{Pl}{AE}$$

in which e = total deformation, in inches
P = force, in pounds
l = length, in inches
A = cross-section area, in square inches
E = modulus of elasticity, in pounds per square inch.

Note that E is expressed in the same unit as f, pounds per square inch, because, in the equation $E = \frac{f}{s}$, s is an abstract number. By referring to Table 2, we find the value of E for certain building materials. The testing laboratories have given us these figures, so that it becomes a simple matter to compute the deformations for direct stresses. The important thing to remember in using this formula is that it is valid only when the unit stress does not exceed the elastic limit of the material. It is noted in Table 2 that $E = 30{,}000{,}000\#/\text{in}^2$ for structural steel. A more exact figure is $29{,}000{,}000\#/\text{in}^2$. The difference is slight, and $30{,}000{,}000\#/\text{in}^2$ is frequently used in computations, especially for the reinforcement in reinforced concrete.

EXAMPLES

EXAMPLE. A 1″ square steel rod, 16″ long, elongates 0.0111″ when subjected to a tensile force of 20,500#. Compute the modulus of elasticity.

SOLUTION. If the area of the rod is 1in^2 and the force is 20,500#, the unit stress $f = 20{,}500\#/\text{in}^2$. Since the unit stress is less than the elastic limit, $35{,}000\#/\text{in}^2$, the formula $E = \frac{Pl}{Ae}$ is valid. Substituting the known values,

$$E = \frac{20{,}500 \times 16}{1 \times 0.0111} = 29{,}549{,}000\#/\text{in}^2$$

EXAMPLE. A steel rod 2″ in diameter and 10′ long is subjected to a tensile force of 60,000#. Compute its deformation.

SOLUTION. A 2″ diameter rod has an area of 3.1416in^2. The unit stress $f = \frac{P}{A} = \frac{60{,}000}{3.1416} = 19{,}108\#$; since this is less than the elastic limit of the material, the formula for finding the deformation is applicable. In the equation $e = \frac{Pl}{AE}$, $P = 60{,}000$, $l = 120$, $A = 3.1416$ and $E = 30{,}000{,}000$ (Table 2). Substituting these values,

$$e = \frac{60{,}000 \times 120}{3.1416 \times 30{,}000{,}000} = 0.076'', \text{ the lengthening of the bar}$$

PROBLEMS

17-A. How much will a ½″ x 3″ wrought iron bar elongate under a tensile force of 20,000# if its length is 20′?

17-B. A 4″ x 3½″ x ½″ steel angle in a truss is 9′ 6″ long and resists a tensile force of 70,000#. How much will it elongate? (Note: For areas of angles see Table 11.)

17-C. A 4″ x 5″ bar of cast iron 20″ long is compressed with a force of 400,000#. How much will it shorten?

17-D. How much will an 8″ x 8″ spruce post, 12′ in length, shorten under an axial load of 45,000#?

17-E. A member of a truss is 12′ in length and is composed of two 4″ x 3″ x ⅜″ angles. If the tensile load on the member is 100,000#, how much will it elongate?

17-F. A ½″ steel cable 100′ long supports a load of 4,000#. How much will it elongate?

17-G. A short 4″ x 4″ cast iron bar 12″ in length is subjected to a compressive load of 500,000#. What is the deformation?

17-H. What force must be applied to a 1″ square steel bar 2′ 0″ in length to produce an elongation of 0.016″?

17-I. A steel column 10′ 0″ in length is subjected to an axial load of 126,000#. If the column is a 6″ WF 22.5# section, how much will it shorten? (See Table 7.)

MOMENTS

18. **Moments.** You have probably often heard the term moment used in connection with problems in engineering; it is of the utmost importance that you understand exactly what the term means. One can readily visualize a force of 100#, an area of 16in^2, or a length of 3′. · A moment, however, is quite different. We define it as *the tendency of a force to cause rotation about a certain point or axis.* It is the result of a force acting on a body in such a manner as to tend to cause the body to rotate about a certain point. *The moment of a force with respect to a given point is the force multiplied by the distance to the point.* The distance, which is called the lever arm, is the distance measured by a line drawn perpendicular to the line of action of the force to the point. The point is called the center of moments. Do not forget that you can never consider the moment of a force without having in mind the

point or axis about which it tends to cause rotation. The units of measurement of a moment are inch-pounds, foot-pounds, and so on, and are written 480,000″# or 40,000′#.

Figure 2 represents two forces acting on a bar which is supported at point A. The moment of the force P_1 about the point A is $8' \times 100\#$, or $800'\#$. This force tends to produce clockwise rotation about the point A (the direction in which the hands of a clock revolve), called a positive moment. The other force P_2 has a lever arm of 4′ about the point A and its moment is $4' \times 200\# = 800'\#$. P_2 tends to rotate the bar in a counterclockwise direction and such a moment is called negative. In this figure, the positive and negative moments are equal, and equilibrium or no motion is the result.

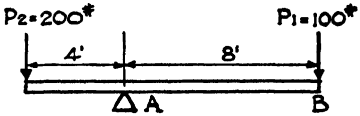


FIG. 2

Three forces, P_1, P_2, and P_3, act on the body shown in Fig. 3. The point A is any point in the plane of the forces, and the forces P_1, P_2, and P_3 have lever arms a, b, and c respectively, with respect to this point. Note that these are the perpendicular distances from the lines of action of the forces to the center of moments, point A.

Forces P_1 and P_2 have positive moments and P_3 a negative moment with respect to the point A. If the three forces and their lever arms are such that equilibrium is produced (that is, that there will be no motion), the sum of the positive moments will equal the sum of the negative moments, or $P_1a + P_2b = P_3c$. Sometimes it is stated thus: If a system of forces is in equilibrium, the algebraic sum of the moments is zero, which in this instance may be written $P_1a + P_2b - P_3c = 0$.

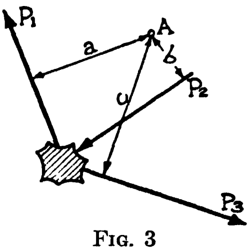


FIG. 3

In Fig. 3 the point A was taken, but the fundamental law holds true for any point that might be selected. In Fig. 2, for instance, the upward force at A equals the sum of the downward forces or $200 + 100 = 300\#$. If we take the sum of the moments about the point B, $200 \times 12 = 300 \times 8$ or $2{,}400'\# = 2{,}400'\#$. Note that the force P_1 about the point B is 100×0 or zero and is therefore omitted from the equation.

Remember, in writing an equation of moments, to be consistent and consider the same center of moments for each force. A moment, since it is a force times a distance, is a compound quantity. If the force is in pounds and the lever arm in inches, the moment is in units of inch-pounds. A moment of 500 foot-pounds of course is equal to 6,000 inch-pounds. This may be written 500'# = 6,000"#.

In problems dealing with beams the forces are parallel and, if there is equilibrium, the sum of the downward forces equals the sum of the upward forces. We may say, *if a system of parallel forces be in equilibrium the algebraic sum of the forces is zero.* Referring to Fig. 2, 200 + 100 − 300 = 0. This law and the law of moments just discussed are in constant application in problems pertaining to beams.

19. **Moments of Forces Acting on a Beam.** Figure 4 (*a*) shows two forces, 100# and 200#, acting on a beam. The beam is 8' between the supports, and the supporting forces, which are called reactions, are 175# and 125#. The four forces are in equilibrium, and therefore the two laws previously mentioned apply. Let us see if this is true.

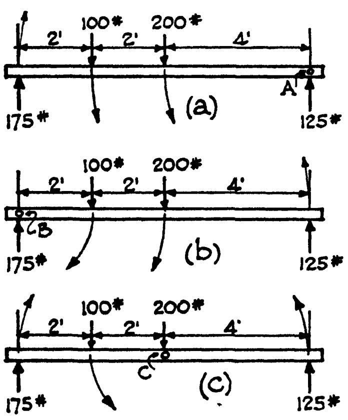


FIG. 4

First, since the forces are in equilibrium, the sum of the downward forces must equal the sum of the upward forces. The sum of the downward forces, the loads, is 100# + 200# = 300#; and the sum of the upward forces, the reactions, is 175# + 125# = 300#. We can write this 100 + 200 = 175 + 125; this is a true statement.

Second, since the forces are in equilibrium, the sum of the moments of the forces tending to cause clockwise rotation (positive moments) must equal the sum of the moments of the forces tending to produce counterclockwise rotation (negative moments) about any center of moments. Let us first write an equation of moments about the point *A*, at the right-hand support. The force tending to cause clockwise rotation about this point is 175#; its

moment is 175×8 or 1,400′#. The forces tending to cause counterclockwise rotation *about the same point* are 100# and 200#, and their moments are (100×6)′# and (200×4)′#. Now that we understand, we can write

$$(175 \times 8) = (100 \times 6) + (200 \times 4)$$
$$1{,}400 = 600 + 800$$
$$1{,}400'\# = 1{,}400'\#$$

which is true.

Perhaps you are wondering what has become of the force 125# in writing this equation. The force 125# has a lever arm of zero feet about the point A, and the moment of the force becomes $125 \times 0 = 0$. Now, in future problems, when we write equations of moments, we can omit writing the moment of the force acting through the point we have selected, because we know it can cause no rotation about this point and its moment is zero.

Let us try again. This time we shall select the point B at the left support as the center of moments, Fig. 4 (*b*). By the same reasoning we can write

$$(100 \times 2) + (200 \times 4) = (125 \times 8)$$
$$200 + 800 = 1{,}000$$
$$1{,}000'\# = 1{,}000'\#$$

Again the law holds. In this case the force 175# has a lever arm of zero feet about the center of moments and its moment is zero.

Suppose we select the point C, Fig. 4 (*c*), as the center of moments; then

$$(175 \times 4) = (100 \times 2) + (125 \times 4)$$
$$700 = 200 + 500$$
$$700'\# = 700'\#$$

We have seen that the law of moments holds in each instance. It is of the utmost importance that we understand this principle thoroughly before going on. Remember that the loads and reactions are usually in units of pounds, and that the moments are compound quantities, foot-pounds or inch-pounds, the result of multiplying a force by a distance.

PROBLEM

19-A. Figure 5 represents a beam in equilibrium with three loads and two reactions. Select five different centers of moments and write the equation of

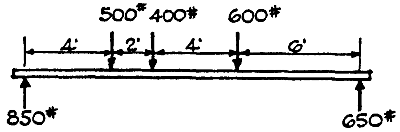


Fig. 5

moments for each, showing that the sum of the positive moments equals the sum of the negative moments.

REACTIONS OF BEAMS

20. Beams. A beam is a structural member, resting upon supports, usually at its ends, which supports transverse loads. The loads acting on a beam tend to *bend* rather than shorten or lengthen it. There are in general five types of beams, the classification depending upon the method and number of supports.

21. Types of Beams. A *simple beam* is a beam which rests upon a support at each end, there being no restraint at the supports. This is the type most commonly in use; see Fig. 6 (*a*).

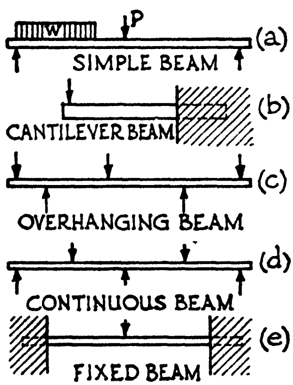


Fig. 6

A *cantilever beam* is a beam that is supported at only one end. A beam projecting from a wall is an example; see Fig. 6 (*b*).

An *overhanging beam* rests upon two or more supports, one or both ends projecting beyond the supports; see Fig. 6 (*c*).

A *continuous beam* is a beam that rests upon more than two supports; see Fig. 6 (*d*).

A *fixed beam* is a beam that is restrained against rotation at its ends; see Fig. 6 (*e*).

22. Kinds of Loads. The two types of loads which commonly occur on beams are called *concentrated* and *distributed*. A concentrated load is assumed to act at a definite point, such as a column resting on a beam. A distributed load is one that acts over a con-

siderable length. A concrete slab resting upon a beam is an illustration of a distributed load. If the distributed load exerts an equal force for each unit of length it is known as a *uniformly distributed load*. Obviously, a distributed load need not extend over the entire length of the beam. Figure 6 (a) illustrates a conventional method of representing the two types of loads: W is the uniformly distributed load, and P is a concentrated load.

23. **Reactions.** Reactions are the upward forces acting at the supports which hold in equilibrium the downward forces or loads.

If we have a beam 18′ in length with a concentrated load of 9,000# at 9′ from the supports, it is readily seen that each support will be equal and will be ½ the load in magnitude, or 4,500#. But consider, for instance, the 9,000# load placed 10′ from one end. What will be the upward supporting forces? Certainly they will not be equal.

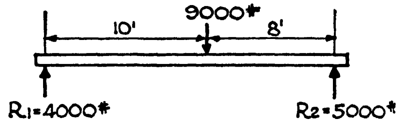


FIG. 7

Now this is where the principle of moments applies. (See Fig. 7.) Let us write an equation of moments, taking the center of moments about the right-hand support, R_2:

$$18R_1 = 9{,}000 \times 8$$

$$R_1 = \frac{72{,}000}{18}$$

$$R_1 = 4{,}000\#$$

Since we know that the sum of the loads is equal to the sum of the reactions we can easily compute R_2, for

$$R_1 + R_2 = 9{,}000$$

$$4{,}000 + R_2 = 9{,}000$$

$$R_2 = 5{,}000\#$$

To check this value of R_2 write an equation of moments about the left-hand support, R_1:

$$18R_2 = 9{,}000 \times 10$$

$$R_2 = \frac{90{,}000}{18}$$

$$R_2 = 5{,}000\#$$

EXAMPLE

Example. A simple beam 20′ in length has three concentrated loads, as indicated in Fig. 8. Compute the magnitude of the reactions.

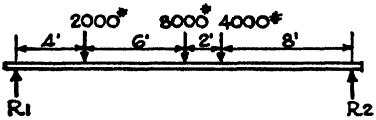


Fig. 8

Solution. With the right-hand support as the center of moments, write the equation of moments. Then

$$20R_1 = (2{,}000 \times 16) + (8{,}000 \times 10) + (4{,}000 \times 8)$$

$$20R_1 = 144{,}000$$

$$R_1 = 7{,}200\#$$

The sum of the reactions equals the sum of the loads:

$$R_1 + R_2 = 2{,}000 + 8{,}000 + 4{,}000$$

$$7{,}200 + R_2 = 14{,}000$$

$$R_2 = 6{,}800\#$$

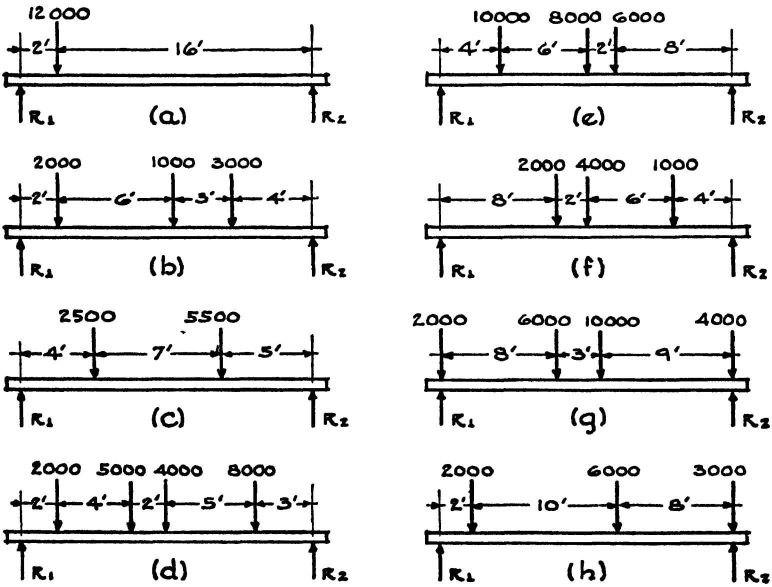


Fig. 9

To check R_2, write the equation of moments about R_1:

$$20R_2 = (2{,}000 \times 4) + (8{,}000 \times 10) + (4{,}000 \times 12)$$

$$20R_2 = 136{,}000$$

$$R_2 = 6{,}800\#$$

PROBLEMS

23-A-B-C-D-E-F-G-H. Eight simple beams with concentrated loads are indicated in Fig. 9. Compute the reactions for each beam and check the results in each case.

24. **Distributed Loads.** So far, we have considered only concentrated loads in computing the magnitudes of reactions. Perhaps you have wondered how we deal with distributed loads. The method is similar to that which has just been described, there being just one thing to remember. *A distributed load on a beam produces the same reactions as a concentrated load of the same magnitude acting through the center of the distributed load.* Now if we bear in mind that the center of gravity of a uniformly distributed load is at the middle of its length, the problem is extremely simple.

EXAMPLES

EXAMPLE. A simple beam 16′ in length has a concentrated load of 8,000# and a uniformly distributed load of 14,000#, as shown in Fig. 10. Compute the reactions.

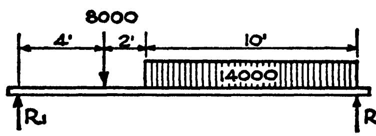


FIG. 10

SOLUTION. Note that the uniformly distributed load extends over a length of 10′. First, let us write an equation of moments about R_2, considering that the uniformly distributed load of 14,000# acts at the middle of its length or 5′ from R_2. Then

$$16R_1 = (8{,}000 \times 12) + (14{,}000 \times 5)$$

$$16R_1 = 166{,}000$$

$$R_1 = 10{,}375\#$$

To find R_2, take the center of moments about R_1. The lever arm of the load 14,000# is 11′ because we consider the uniformly distributed load as acting at its mid-point or 11′ from R_1. Then

$$16R_2 = (8{,}000 \times 4) + (14{,}000 \times 11)$$

$$16R_2 = 186{,}000$$

$$R_2 = 11{,}625\#$$

To check these results, the sum of the loads should equal the sum of the reactions, or

$$8{,}000 + 14{,}000 = 10{,}375 + 11{,}625$$

$$22{,}000\# = 22{,}000\#$$

EXAMPLE. Compute the reactions for the beam loaded as shown in Fig. 11.

SOLUTION. This beam has a uniformly distributed load of 20,000# extending over a length of 8′; therefore its mid-point is 6′ from R_1 and 8′ from R_2. Writing an equation of moments about R_2,

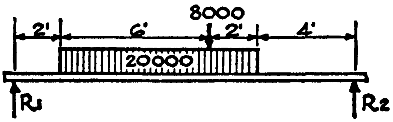


FIG. 11

$$14R_1 = (20{,}000 \times 8) + (8{,}000 \times 6)$$

$$14R_1 = 160{,}000 + 48{,}000$$

$$R_1 = 14{,}857.1\#$$

Taking R_1 as the center of moments,

$$14R_2 = (20{,}000 \times 6) + (8{,}000 \times 8)$$

$$14R_2 = 120{,}000 + 64{,}000$$

$$R_2 = 13{,}142.9\#$$

Check:

$$20{,}000 + 8{,}000 = 14{,}857.1 + 13{,}142.9$$

$$28{,}000\# = 28{,}000\#$$

PROBLEMS

24-A-B-C-D-E-F-G-H-I-J. Compute the reactions for the ten beams shown in Fig. 12, and check the results.

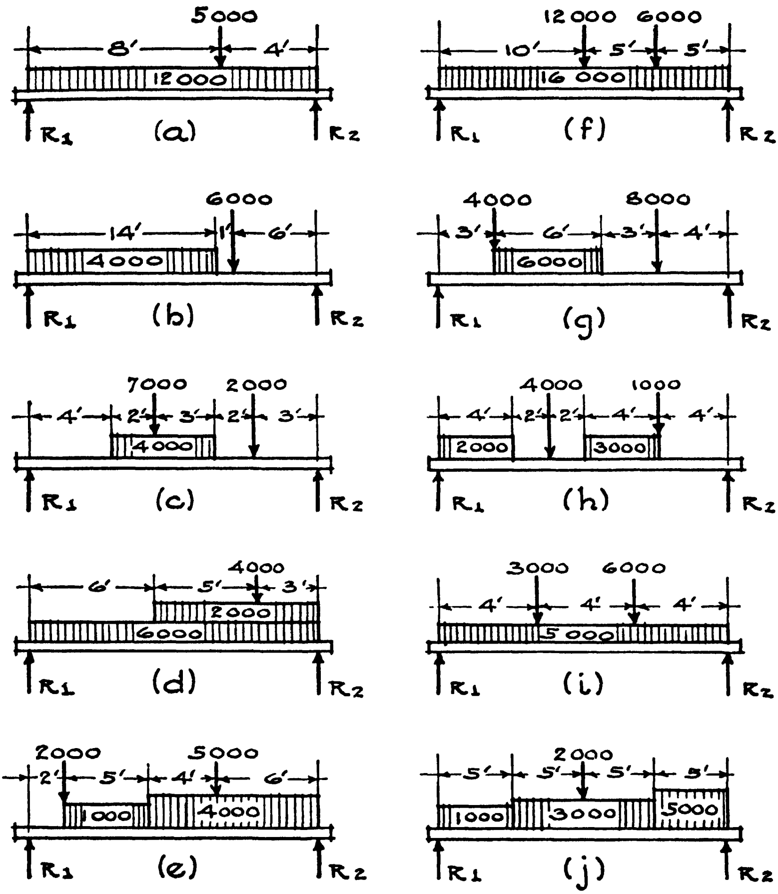


FIG. 12

25. **Overhanging Beams.** The method of computing reactions for overhanging beams is exactly the same method employed in the previous examples. Select one of the reactions as a center of moments and write the equation of moments. On one side of the equation place the sum of the moments tending to cause clockwise rotation, and on the other side the sum of the moments tending to cause rotation in the opposite direction. Bear two points in mind: first, *be consistent and take the same center of moments for*

each force; and, second, *consider uniformly distributed loads as acting at their mid-points.*

EXAMPLES

EXAMPLE. Compute the reactions of the overhanging beam shown in Fig. 13 (*a*), and check the results.

SOLUTION. Select R_1 as the center of moments. The forces tending to cause clockwise rotation *about this point* are 4,000#,

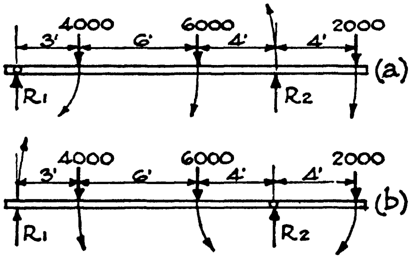


FIG. 13

6,000#, and 2,000#, and the only force tending to cause counterclockwise rotation is R_2. Therefore

$$13R_2 = (4{,}000 \times 3) + (6{,}000 \times 9) + (2{,}000 \times 17)$$

$$13R_2 = 12{,}000 + 54{,}000 + 34{,}000$$

$$R_2 = 7{,}692.3\#$$

Now to find R_1 we must take the center of moments about R_2. The forces tending to cause clockwise rotation *about this point* are R_1 and 2,000#, and those which tend to cause counterclockwise rotation *about the same point* are 4,000# and 6,000#. See Fig. 13 (*b*). Then

$$13R_1 + (2{,}000 \times 4) = (4{,}000 \times 10) + (6{,}000 \times 4)$$

$$13R_1 = 40{,}000 + 24{,}000 - 8{,}000$$

$$13R_1 = 56{,}000$$

$$R_1 = 4{,}307.7\#$$

To check these results, the sum of the loads should equal the sum of the reactions; therefore

$$4{,}000 + 6{,}000 + 2{,}000 = 4{,}307.7 + 7{,}692.3$$
$$12{,}000\# = 12{,}000\#$$

EXAMPLE. The overhanging beam shown in Fig. 14 has a concentrated load of 4,000# and a uniformly distributed load of 6,000#. Compute the reactions.

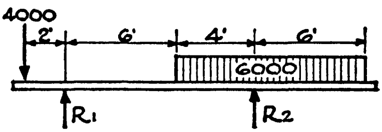


FIG. 14

SOLUTION. This beam overhangs both ends. Note that the uniformly distributed load extends over a length of 10′ and therefore it may be considered as acting at its mid-point, which is 1′ to the right of R_2. Taking the center of moments about R_1,

$$10R_2 + (4{,}000 \times 2) = (6{,}000 \times 11)$$
$$10R_2 = 66{,}000 - 8{,}000$$
$$R_2 = 5{,}800\#$$

Taking the center of moments about R_2,

$$10R_1 + (6{,}000 \times 1) = (4{,}000 \times 12)$$
$$10R_1 = 48{,}000 - 6{,}000$$
$$R_1 = 4{,}200\#$$

Check:

$$4{,}000 + 6{,}000 = 5{,}800 + 4{,}200$$
$$10{,}000\# = 10{,}000\#$$

PROBLEMS

25-A-B-C-D-E-F-G-H-I-J. Compute the reactions and check the results for the beams shown in Fig. 15.

WARNING! The importance of one's ability to compute the magnitudes of the reactions of beams cannot be stressed too highly. The principle involved occurs time after time in engineering problems; therefore it is necessary to understand thoroughly the fore-

going paragraphs before proceeding. Make some problems for yourself to test your knowledge. It is a simple matter to test the results for, after you compute the reactions by the principle of moments, remember that their sum must equal the sum of the loads.

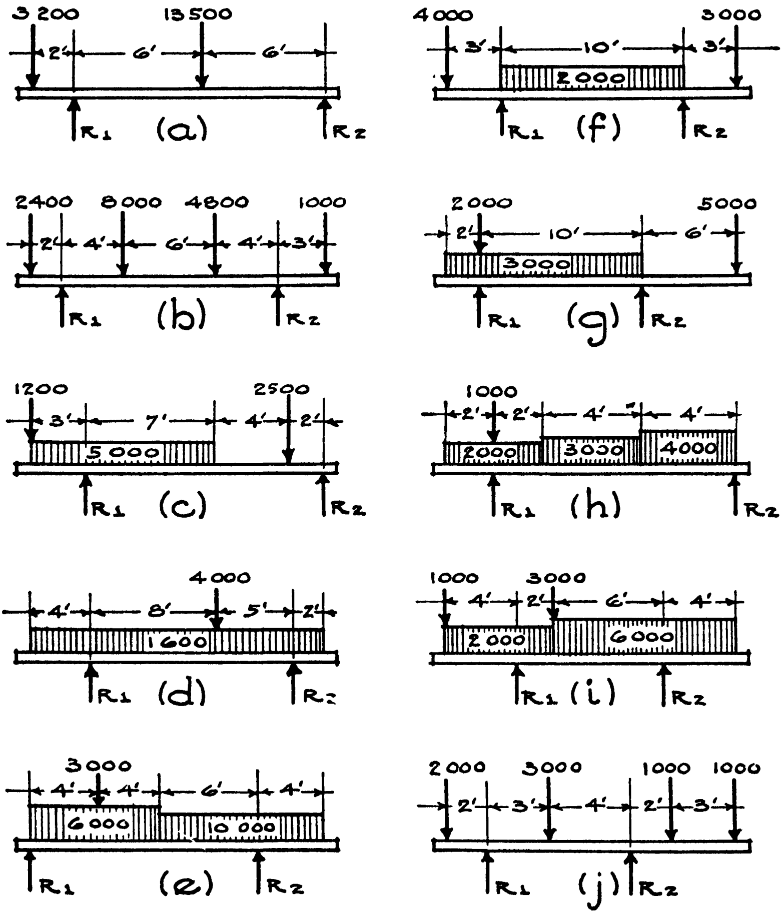


Fig. 15

SHEAR AND BENDING MOMENTS

26. Stresses in Beams. Figure 16 (*a*) represents a simple beam having a uniformly distributed load *W* over its entire length. Examining a practical example would probably reveal no effects

of the load on the beam. However, there are three distinct major tendencies for the beams to fail. Figure 16 (*b*), 16 (*c*), and 16 (*d*) are, of course, exaggerations, but they illustrate the three phenomena. First, there is a tendency for the beam to fail by dropping down between the supports. This is called *vertical shear;* see Fig. 16 (*b*). Second, the beam may fail by *bending;* see Fig. 16 (*c*). Third, there is a tendency for the fibers of the beam to slide past each other in a horizontal direction; see Fig. 16 (*d*). The name given to this is *horizontal shear*, and a brief discussion concerning it will be found under Art. 47. Naturally, a beam properly selected does not fail in any of the ways just mentioned, but the tendencies to fail exist nevertheless. The forces which prevent failure are supplied by the fibers in the beam. They are resisting stresses, and our problem is to select beams whose dimensions will provide adequate material to supply these resisting stresses.

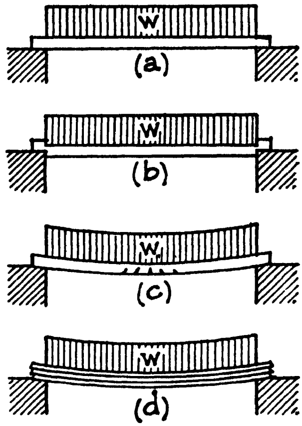


FIG. 16

27. **Vertical Shear.** We can define vertical shear as the tendency for one part of a beam to move vertically with respect to an adjacent part. *Its magnitude at any section of a beam is equal to the algebraic sum of the vertical forces on either side of the section.* It is usually represented by the letter V. For convenience in our problems, we shall consider the forces to the left of the section, remembering, however, that the same result will be obtained if we take those on the right. We may say then that *the vertical shear at any section of a beam is equal to the reactions minus the loads to the left of the section.* Do not forget this. Now, if we wish to find the magnitude of the vertical shear at any section of a beam, we simply repeat the above statement and write an equation accordingly. From this it is seen that the maximum shear for simple beams is equal to the greatest reaction, and for symmetrically loaded beams the maximum shear equals ½ the total load. See Figs. 34 and 35.

If the loads and reactions are in units of pounds, as they usually are, then the magnitude of the vertical shear will be in units of

pounds also. Form the habit of writing the denomination of the units; it will prevent many errors.

EXAMPLES

EXAMPLE. Figure 17 (*a*) indicates a simple beam having two concentrated loads of 600# and 1,000#. Our problem will be to find the value of the vertical shear at various points in the length of the beam. The weight of the beam constitutes a uniformly distributed load. It will be ignored in this problem.

SOLUTION. The reactions are computed by the principle of moments as previously described, and we find them to be R_1 =

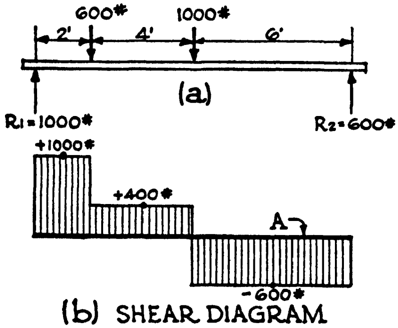


FIG. 17

1,000# and R_2 = 600#. Consider first the value of the vertical shear, V, at an infinitely short distance to the right of R_1. Applying the rule that the shear is equal to the reactions minus the loads to the left of the section, we write, $V = R_1 - 0$, or V = 1,000#. The zero represents the value of the loads to the left of the section which, of course, is zero. Now take a section 1′ to the right of R_1; again $V_{(x=1)} = R_1 - 0$, or $V_{(x=1)}$ = 1,000#. The subscript ($x = 1$) indicates the position of the section at which the shear is taken. We find that the shear is still 1,000#, and the shear has the same magnitude up to the 600# load. The next section to consider will be at a very short distance to the right of the load 600#. Then $V_{(x=2+)}$ = 1,000 − 600 = 400#. Since there are no loads intervening, the shear continues to be the same magnitude up to the load 1,000#. At a section a short distance to the right

of the 1,000# load, $V_{(x=6+)} = 1{,}000 - (600 + 1{,}000) = -600\#$. This magnitude continues up to the right-hand reaction R_2.

The previous example dealt only with concentrated loads. Let us see if we can apply the same principles to a beam having a uniformly distributed load in addition to a concentrated load.

EXAMPLE. Compute the value of the vertical shear at various sections of the beam shown in Fig. 18 (*a*).

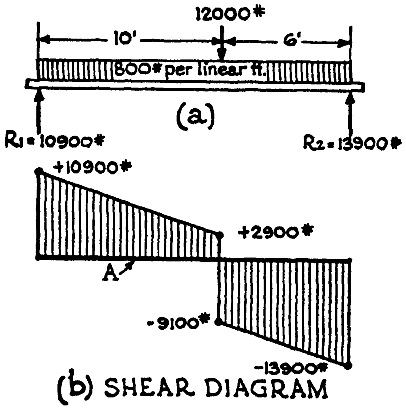


FIG. 18

SOLUTION. Note that there is one concentrated load of 12,000# at 6′ from R_2 and also a uniformly distributed load of 800# *per linear foot* over the entire length of the beam. When the load per foot is given it is usually represented as $w\#$, the capital letter, W, being used to represent the *total* uniformly distributed load. In this instance $w = 800\#$ and $W = 800 \times 16 = 12{,}800\#$.

Writing the equation of moments about R_2 as the center,

$$16R_1 = (800 \times 16 \times 8) + (12{,}000 \times 6)$$

$$16R_1 = 102{,}400 + 72{,}000$$

$$R_1 = \frac{174{,}400}{16} = 10{,}900\#$$

In a similar manner R_2 is found to be 13,900#. In the quantity $(800 \times 16 \times 8)$ the load $(800 \times 16)\#$ has a lever arm of 8′, the distance of its center of gravity to the reaction.

Following the rule used in the previous example, write the value of V at various sections in the beam:

$$V_{(x=0)} = 10{,}900 - 0 = 10{,}900\#$$

$$V_{(x=1)} = 10{,}900 - 800 = 10{,}100\#$$

$$V_{(x=5)} = 10{,}900 - (800 \times 5) = 6{,}900\#$$

$$V_{(x=10-)} = 10{,}900 - (800 \times 10) = 2{,}900\#$$

$$V_{(x=10+)} = 10{,}900 - [(800 \times 10) + 12{,}000] = -9{,}100\#$$

$$V_{(x=16)} = 10{,}900 - [(800 \times 16) + 12{,}000] = -13{,}900\#$$

Note that the value of the vertical shear at the supports is the same magnitude as the reactions.

28. **Shear Diagrams.** In the two preceding examples we have computed the value of the shear at several sections in the length of the beams, and you probably wonder why all this bother. Has it any practical value? It most certainly has, as we shall see presently. In order to visualize the results we have obtained, suppose we make diagrams showing these values. These are called shear diagrams, and by inspecting them we are enabled to establish directly two very significant facts.

To make such a diagram, first draw the beam to scale and locate the loads. Beneath the beam draw a horizontal line such as A, Fig. 17 (*b*) and Fig. 18 (*b*). Above and below this line as a base, plot at any convenient scale the values of the shear at the various sections; the positive or plus values are placed above the line and the negative or minus values below. In Fig. 17, for instance, the value of the shear at R_1 is $+1{,}000\#$. The shear continues to be the same value up to the load of 600#, at which point it drops to $+400\#$. The same value continues up to the next load, 1,000#, where it drops to $-600\#$ and continues to the right-hand reaction. Obviously, to draw a shear diagram it is necessary to compute the values at significant points only. Having made the diagram, we may readily find the value of the shear by scaling the vertical distance in the diagram at any section of the beam. The shear diagram for the beam in Fig. 18 is made in the same manner.

There are two important facts to note concerning the vertical shear. The first is the maximum value. We see from the diagrams

just made that in each case the maximum shear is at the reaction having the greatest value, and its magnitude is the magnitude of the greatest reaction. In Fig. 17, the maximum shear is 1,000#,

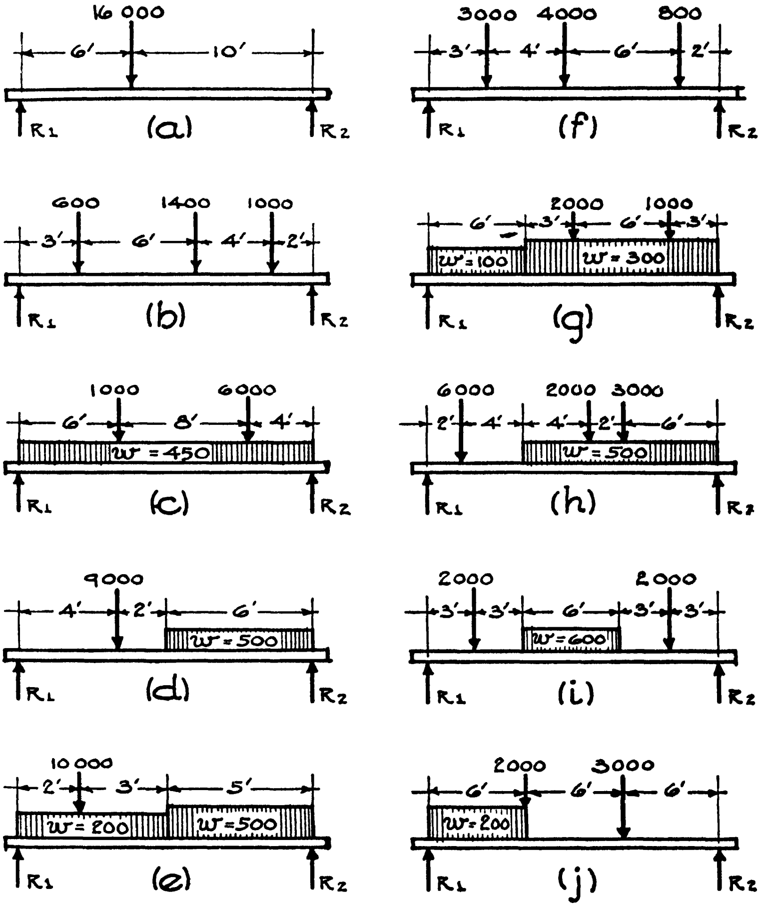


FIG. 19

and in Fig. 18 it is 13,900#. We ignore the positive or negative signs in reading the maximum values of the shear since the diagrams are merely conventional methods of representing the values.

The other important fact is to note the point at which the shear changes from a plus to a minus quantity. We say, "the point at

which the shear passes through zero." In Fig. 17 it is under the 1,000# load, at 6′ from R_1. In Fig. 18 the shear passes through zero at 10′ from R_1. The reason for finding this point is that *the greatest tendency for the beam to fail by bending is at the point where the shear passes through zero.*

PROBLEMS

28-A-B-C-D-E-F-G-H-I-J. For the beams indicated in Fig. 19 draw the shear diagrams and note in each instance the value of the maximum shear and the point at which the shear passes through zero. Notice that the distributed loads are represented by w, which means so many pounds per foot.

29. Bending Moments. The forces which tend to cause bending in a beam are the reactions and the loads. Consider the section X–X at 6′ from R_1 (Fig. 20). The force R_1 or 2,000# tends to cause a clockwise rotation about this point. Since the force is 2,000# and the lever arm is 6′, the moment of the force is $2{,}000 \times 6 = 12{,}000'\#$. This same value may be found by considering the

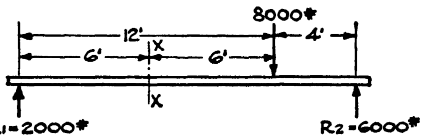


FIG. 20

forces to the right of the section X–X. Let us see. There are two forces to the right of section X–X; R_2, which is 6,000#, and the load 8,000#, having lever arms of 10′ and 6′ respectively. The moment of the reaction is $6{,}000 \times 10 = 60{,}000'\#$, and its direction is counterclockwise with respect to the section. The moment of the force 8,000# is $8{,}000 \times 6 = 48{,}000'\#$, and its direction is clockwise. Subtracting, $60{,}000'\# - 48{,}000'\# = 12{,}000'\#$, the resultant moment tending to cause counterclockwise rotation about the section X–X. This is the same magnitude as the moment of the forces on the left which tends to cause a clockwise rotation.

Thus it makes no difference whether we consider the forces to the right of the section or the left; the magnitude of the moment is the same. It is called the bending moment and it *is the algebraic sum of the moments of the forces on either side of the section.* For simplicity, let us take them on the left; then we may say *the bend-*

ing moment at any section of a beam is equal to the moments of the reactions minus the moments of the loads to the left of the section. Since the bending moment is the result of multiplying forces by distances, the denominations are foot-pounds, inch-pounds, and so on.

Almost everyone confuses shear and bending moment at first. Remember that the shear is the result of reactions minus loads, the units being usually pounds, but the bending moment is the result of subtracting *moments* of loads from *moments* of reactions, and the units are usually foot-pounds or inch-pounds.

30. **Bending-Moment Diagrams.** Bending-moment diagrams are drawn in the same manner as shear diagrams. A horizontal line, having the same length as the beam, is drawn below the beam, usually under the shear diagram. Then the bending moments are computed for various sections of the beam and the vertical distances are plotted to scale. In simple beams all the moments are positive and, therefore, are plotted above the base line. In overhanging or continuous beams, we shall find negative moments, and they will be placed below.

EXAMPLE

EXAMPLE. Figure 21 shows a simple beam with two concentrated loads. Draw the shear and moment diagrams.

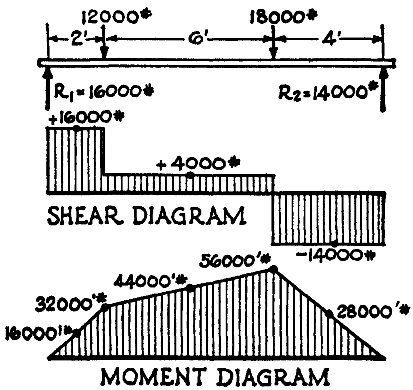


FIG. 21

SOLUTION. R_1 and R_2 are first computed and are found to be 16,000# and 14,000# respectively. Next the shear diagram is drawn

as described in Art. 28. Note that it is necessary to compute the shear at only one section because there is no distributed load and we know the value of the shear at the reactions to be equal to the reactions in magnitude.

Since the value of the bending moment at any section of the beam is equal to the moments of the reactions minus the moments of the loads to the left of the section, the moment at R_1 must be zero since there are no forces to the left, or we might say that R_1 has a lever arm of zero and $R_1 \times 0 = 0$. Other values in the length of the beam are computed as follows.

$$M_{(x=1)} = (16{,}000 \times 1) = 16{,}000'\#$$

$$M_{(x=2)} = (16{,}000 \times 2) = 32{,}000'\#$$

$$M_{(x=5)} = (16{,}000 \times 5) - (12{,}000 \times 3) = 44{,}000'\#$$

$$M_{(x=8)} = (16{,}000 \times 8) - (12{,}000 \times 6) = 56{,}000'\#$$

$$M_{(x=10)} = (16{,}000 \times 10) - [(12{,}000 \times 8) + (18{,}000 \times 2)] = 28{,}000'\#$$

$$M_{(x=12)} = (16{,}000 \times 12) - [(12{,}000 \times 10) + (18{,}000 \times 4)] = 0$$

The result of plotting these points is shown in the bending-moment diagram (Fig. 21). More points were computed in this instance than were actually necessary. We know the moments at the supports of simple beams to be zero, and only the moments directly under the loads were needed.

31. **Importance of Shear and Moment Diagrams.** In simple beams the value of the vertical shear is zero or passes through zero at some point between the supports. An important principle in this respect is that the *bending moment is a maximum wherever the shear passes through zero.* In Fig. 21 the shear passed through zero under the 18,000# load, that is, at ($x = 8$). Notice that the bending moment has its greatest value at this same point, 56,000'#. In order to design beams, we must know the value of the maximum bending moment. Frequently, we draw only enough of the shear diagram to find the section at which the shear passes through zero and then compute the bending moment at this point. Bending-moment diagrams are of particular assistance in designing plate girders and reinforced concrete beams, as will be shown later.

EXAMPLES

EXAMPLE. Draw the shear and moment diagrams for the beam shown in Fig. 22, having a uniformly distributed load of 400 pounds per linear foot and a concentrated load of 21,000# at 4′ from R_1.

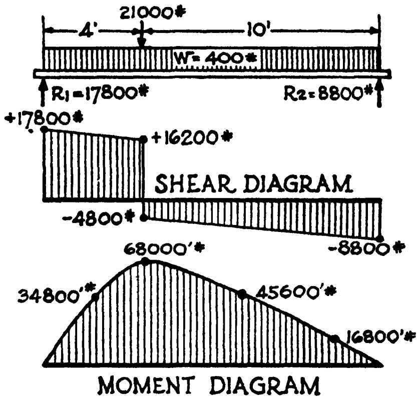


FIG. 22

SOLUTION. Solving the reactions,

$$14R_1 = (21{,}000 \times 10) + (400 \times 14 \times 7)$$

$$R_1 = 17{,}800\#$$

$$\text{Total load} = 21{,}000 + (400 \times 14) = 26{,}600\#$$

Therefore

$$R_1 + R_2 = 26{,}600$$

$$17{,}800 + R_2 = 26{,}600$$

$$R_2 = 26{,}600 - 17{,}800 = 8{,}800\#$$

Computing the value of the shear at essential points,

$$V_{(\text{at } R_1)} = 17{,}800\#$$

$$V_{(x=4-)} = 17{,}800 - (400 \times 4) = 16{,}200\#$$

$$V_{(x=4+)} = 17{,}800 - [(400 \times 4) + 21{,}000] = -4{,}800\#$$

$$V_{(\text{at } R_2)} = -8{,}800\#$$

Note that the shear passes through zero under the 21,000# load; therefore at this point we shall find the bending moment to be a maximum.

Computing the value of the bending moment,

$$M_{(x=0)} = 0$$

$$M_{(x=2)} = (17{,}800 \times 2) - (400 \times 2 \times 1) = 34{,}800'\#$$

In the above equation the reaction to the left of the section is 17,800# and its lever arm is 2′. The load to the left of the section is (400 × 2)# and its lever arm is 1′, the distance from the center of gravity of the load, (400 × 2)#, to the center of moments.

$$M_{(x=4)} = (17{,}800 \times 4) - (400 \times 4 \times 2) = 68{,}000'\#$$

$$M_{(x=8)} = (17{,}800 \times 8) - [(400 \times 8 \times 4) + (21{,}000 \times 4)] = 45{,}600'\#$$

$$M_{(x=12)} = (17{,}800 \times 12) - [(400 \times 12 \times 6) + (21{,}000 \times 8)] = 16{,}800'\#$$

$$M_{(x=14)} = (17{,}800 \times 14) - [(400 \times 14 \times 7) + (21{,}000 \times 10)] = 0$$

From the two previous examples (Figs. 21 and 22) it will be observed that the shear for the parts of the beam where no loads occur is represented by horizontal lines, and that for the parts where a uniformly distributed load occurs by straight inclined lines. The bending moment is represented by straight inclined lines where only concentrated loads occur and by a curved line if the load is distributed.

Occasionally, when a beam has both concentrated and uniformly distributed loads, the shear does not pass through zero under one of the concentrated loads. This frequently occurs when the distributed load is relatively large compared with the concentrated loads. Since it is necessary in designing beams to find the maximum bending moment, we must know the point where it occurs. This, of course, is the point where the shear passes through zero. To find this point is remarkably simple.

EXAMPLE. Compute the maximum bending moment for the beam shown in Fig. 23.

SOLUTION. R_1 and R_2 are computed and found to be 10,600# and 7,600# respectively. When the shear diagram is drawn (Fig. 23) we observe that the shear passes through zero at some point between the concentrated load of 7,000# and the right reaction. Call this distance x feet from R_1. Now we know that the value of the shear at this section is zero; therefore we write the value of the shear for this point, using the terms of the reaction and loads, and equate the quantity to zero.

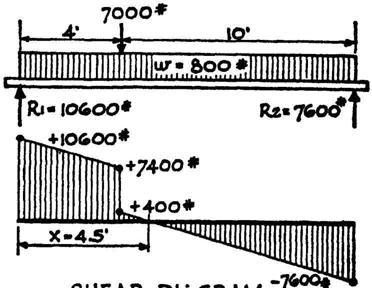


FIG. 23

This equation will contain the distance x, thus:

$$V_{(\text{at } x)} = 10{,}600 - [7{,}000 + (800x)] = 0$$

$$800x = 3{,}600$$

$$x = 4.5'$$

To find the value of the bending moment at this section, we write

$$M_{(x=4.5)} = (10{,}600 \times 4.5) - \left[(7{,}000 \times 0.5) + \left(800 \times 4.5 \times \frac{4.5}{2}\right)\right]$$

$$M = 36{,}100'\#$$

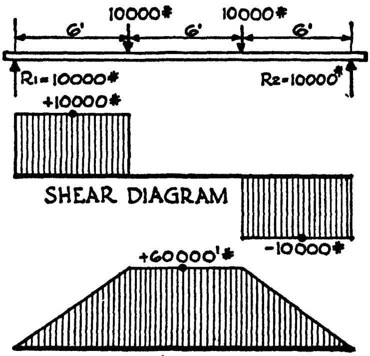


FIG. 24

Under certain conditions the value of the shear is zero for the entire distance between concentrated loads. This occurs when a simple beam has two equal loads at equal distances from the supports. The value of the bending moment is the same at any section between the loads.

EXAMPLE. Two loads of 10,000# each are located at 6′ from the supports (Fig. 24). Compute the maximum bending moment.

SOLUTION. Since the beam is symmetrically loaded, each reaction is equal to ½ the total load or ½ × 20,000 = 10,000#. The

shear diagram is readily drawn as shown. Note that the shear has a zero value at all points between the loads. According to the rule, the bending moment must be a maximum over this portion of the beam. Compute M at various sections, and plot the diagram.

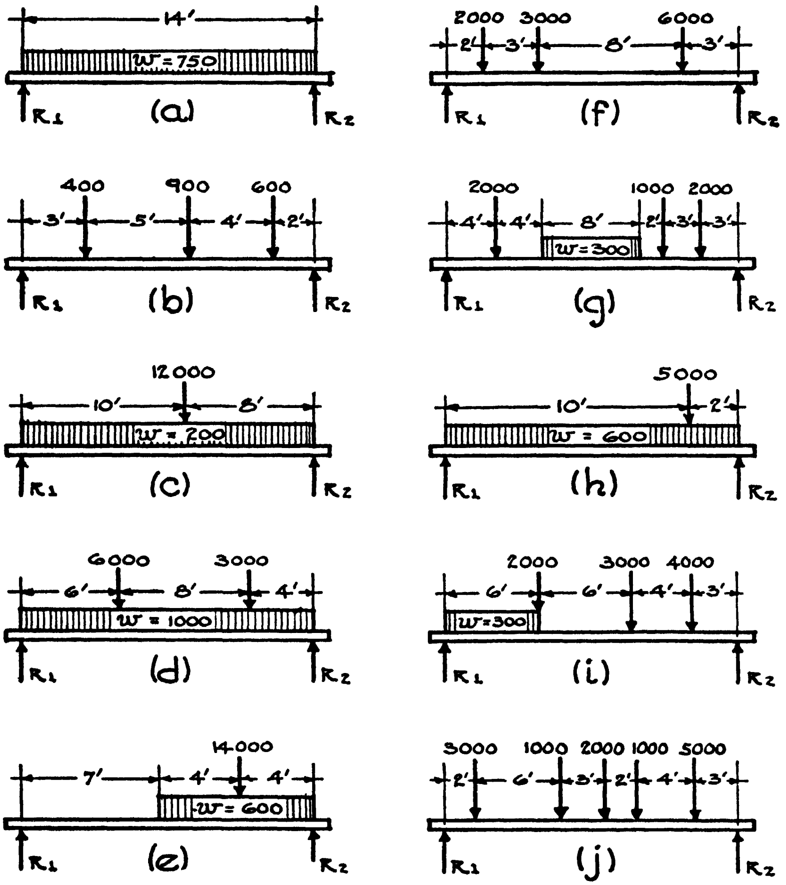


FIG. 25

$$M_{(x=6)} = 10{,}000 \times 6 = 60{,}000'\#$$

$$M_{(x=9)} = (10{,}000 \times 9) - (10{,}000 \times 3) = 60{,}000'\#$$

When such a problem occurs we know, therefore, that the maximum bending moment is equal to one of the loads multiplied by its distance to the nearest reaction.

PROBLEMS

31-A-B-C-D-E-F-G-H-I-J. Draw the shear and moment diagrams for the beams shown in Fig. 25. In each case note the maximum shear and the maximum bending moment.

32. Negative Bending Moments. Overhanging Beams. When a simple beam bends, there is a tendency to assume the shape

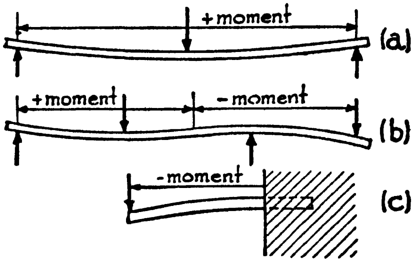


FIG. 26

shown in Fig. 26 (*a*). In this case the fibers in the upper part of the beam are in compression and the bending moment is considered positive. Another way to describe a positive bending moment is to say that the bending moment is positive when the curve assumed by a bent beam is concave upward. When a beam projects beyond a support, Figs. 26 (*b*) and 26 (*c*), a portion of the beam has tensile stresses in its upper part, and the bending moment for this condition is called negative; the beam is bent concave downward. If we construct moment diagrams, following the method previously described, the positive and negative moments are shown graphically.

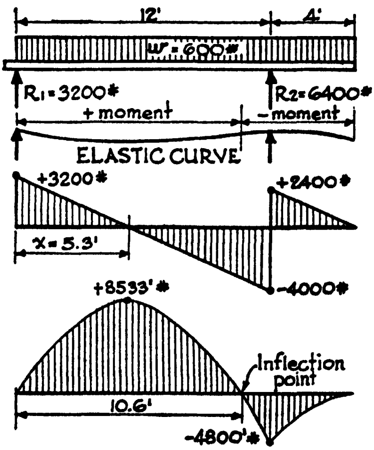


FIG. 27

EXAMPLES

EXAMPLE. Draw the shear and moment diagrams for the overhanging beam shown in Fig. 27.

SOLUTION. Solving the reactions,

$$12R_2 = 600 \times 16 \times 8 \qquad R_2 = 6{,}400\#$$

$$12R_1 = 600 \times 16 \times 4 \qquad R_1 = 3{,}200\#$$

Computing the value of the shear,

$$V_{(\text{at } R_1)} = +3{,}200\#$$

$$V_{(x=12-)} = 3{,}200 - (600 \times 12) = -4{,}000\#$$

$$V_{(x=12+)} = (3{,}200 + 6{,}400) - (600 \times 12) = +2{,}400\#$$

$$V_{(x=16)} = (3{,}200 + 6{,}400) - (600 \times 16) = 0$$

Finding the position where the shear passes through zero between the supports (see example under Art. 31),

$$3{,}200 - 600x = 0$$

$$x = 5.3'$$

Computing the values of the bending moment,

$$M_{(\text{at } R_1)} = 0$$

$$M_{(x=5.3)} = (3{,}200 \times 5.3) - \left(600 \times 5.3 \times \frac{5.3}{2}\right) = 8{,}533'\#$$

$$M_{(x=12)} = (3{,}200 \times 12) - (600 \times 12 \times 6) = -4{,}800'\#$$

$$M_{(x=16)} = 0$$

It is seen on plotting these values that there are two points at which the shear passes through zero, at $x = 5.3'$ and at $x = 12'$. The bending-moment diagram shows that maximum values are found at each point, one being a positive moment and the other a negative moment. When we design beams we are concerned only with the maximum value, regardless of whether it is positive or negative. The shear diagram does not indicate which of the two points gives the greater value of the bending moment, and often it is necessary to compute the value at each point where the shear passes through zero to determine which is the greatest numerically. For this beam it is 8,533'#.

The bending-moment diagram indicates one point between the supports at which the value of $M = 0$. This is called the *inflection point;* it is the point where the curvature reverses, changing from

concave to convex. It is of importance to know the position of the inflection point in the study of reinforced concrete beams, for this is the position where the tensile steel reinforcement is bent upward.

In this problem, call x the distance from the left support to the point where $M = 0$. Then, writing an equation for the value of the bending moment and equating it to zero,

$$(3{,}200 \times x) - \left(600 \times x \times \frac{x}{2}\right) = 0$$

$$3{,}200x - 300x^2 = 0$$

$$x = 10.6'$$

EXAMPLE. Compute the maximum bending moment for the overhanging beam shown in Fig. 28.

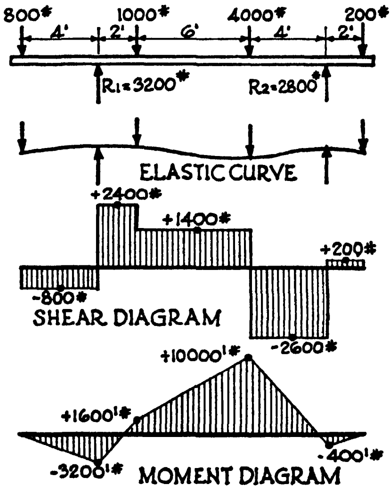


FIG. 28

SOLUTION. Computing the reactions,

$$12R_1 + (200 \times 2) = (800 \times 16) + (1{,}000 \times 10) + (4{,}000 \times 4)$$

$$R_1 = 3{,}200\#$$

$$\text{Total load} = 800 + 1{,}000 + 4{,}000 + 200 = 6{,}000\#$$

Therefore

$$R_2 = 6{,}000 - 3{,}200 = 2{,}800\#$$

Computing the values of the shear,

$V_{(x=1)} \quad = -800\#$

$V_{(x=4+)} \quad = 3{,}200 - 800 = +2{,}400\#$

$V_{(x=6+)} \quad = 3{,}200 - (800 + 1{,}000) = +1{,}400\#$

$V_{(x=12+)} = 3{,}200 - (800 + 1{,}000 + 4{,}000) = -2{,}600\#$

$V_{(x=16+)} = (3{,}200 + 2{,}800) - (800 + 1{,}000 + 4{,}000) = +200\#$

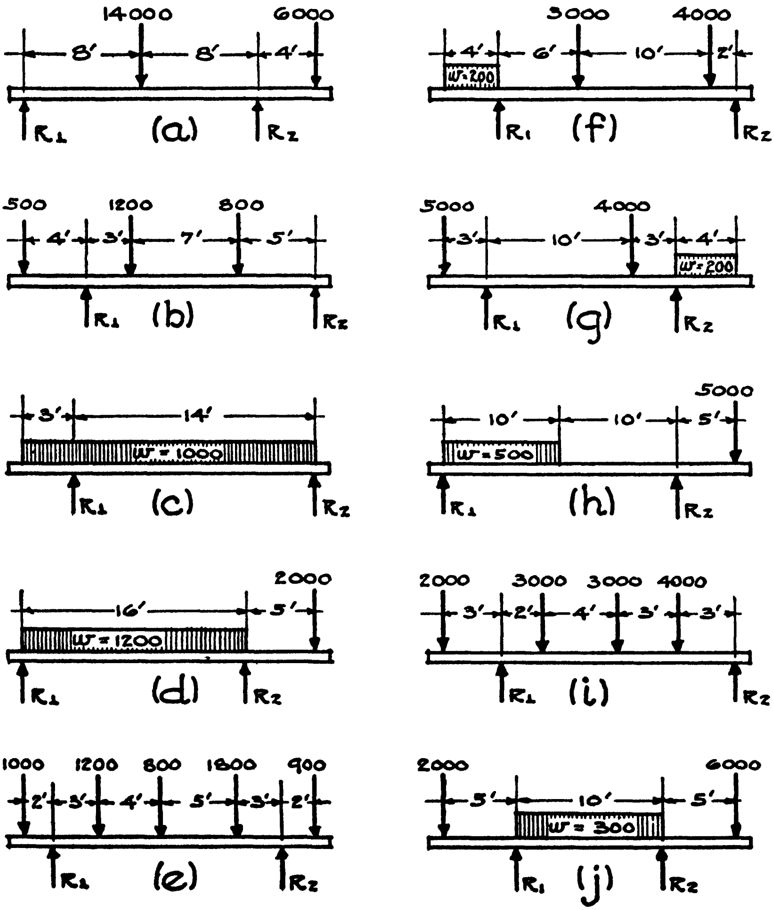


FIG. 29

Plot the shear diagram and note that the shear passes through zero at R_1, R_2, and under the 4,000# load. We shall expect the bending moment to have maximum values at these points.

Computing the values of the bending moments,

$$M_{(x=0)} = 0$$

$$M_{(x=4)} = -(800 \times 4) = -3{,}200'\#$$

$$M_{(x=6)} = (3{,}200 \times 2) - (800 \times 6) = +1{,}600'\#$$

$$M_{(x=12)} = (3{,}200 \times 8) - [(800 \times 12) + (1{,}000 \times 6)] = +10{,}000'\#$$

$$M_{(x=16)} = (3{,}200 \times 12) - [(800 \times 16) + (1{,}000 \times 10) + (4{,}000 \times 4)] = -400'\#$$

$$M_{(x=18)} = 0$$

The maximum value of the bending moment is 10,000'#, occurring under the 4,000# load.

PROBLEMS

32-A-B-C-D-E-F-G-H-I-J. Draw shear and bending-moment diagrams for the beams indicated in Fig. 29. Determine the maximum bending moment in each case.

33. **Cantilever Beams.** In computing the shear and bending moments for cantilever beams it is most convenient to consider the forces between the section considered and the free end. In other words, draw the beam with the fixed end at the right and compute the shear and moments by considering the forces to the left, as has been done in the previous examples.

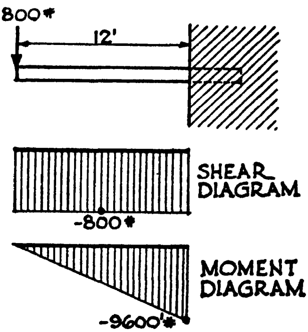


FIG. 30

EXAMPLES

EXAMPLE. The cantilever beam shown in Fig. 30 is 12' in length and has a concentrated load of 800# at the free end. Draw the shear and moment diagrams. What are the values of the maximum shear and maximum bending moment?

Solution. The value of the shear is −800# throughout the entire length of the beam. The bending moment is maximum at the wall; its value is −9,600′#.

$$M_{(x=0)} = 0$$

$$M_{(x=1)} = -(800 \times 1) = -800'\#$$

$$M_{(x=2)} = -(800 \times 2) = -1{,}600'\#$$

$$M_{(x=12)} = -(800 \times 12) = -9{,}600'\#$$

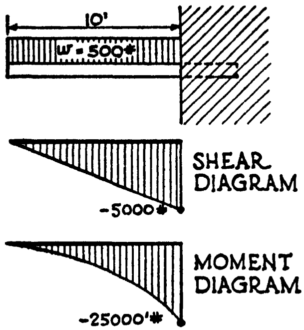


Fig. 31

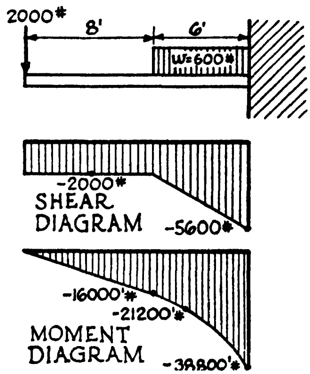


Fig. 32

Example. Draw shear and moment diagrams for a cantilever beam 10′ in length having a uniformly distributed load of 500 pounds per linear foot (Fig. 31).

Solution. Computing the value of the shear,

$$V_{(x=1)} = -(500 \times 1) = -500\#$$

$$V_{(x=2)} = -(500 \times 2) = -1{,}000\#$$

$$V_{(x=10)} = -(500 \times 10) = -5{,}000\#, \text{ the maximum value}$$

Computing the moments,

$$M_{(x=2)} = -(500 \times 2 \times 1) = -1{,}000'\#$$

$$M_{(x=4)} = -(500 \times 4 \times 2) = -4{,}000'\#$$

$$M_{(x=10)} = -(500 \times 10 \times 5) = -25{,}000'\#, \text{ the maximum value}$$

Example. The cantilever beam indicated in Fig. 32 has a concentrated load of 2,000# and a uniformly distributed load of 600#/ft

at the positions shown. Draw the shear and moment diagrams. What are the maximum shear and the maximum bending moment?

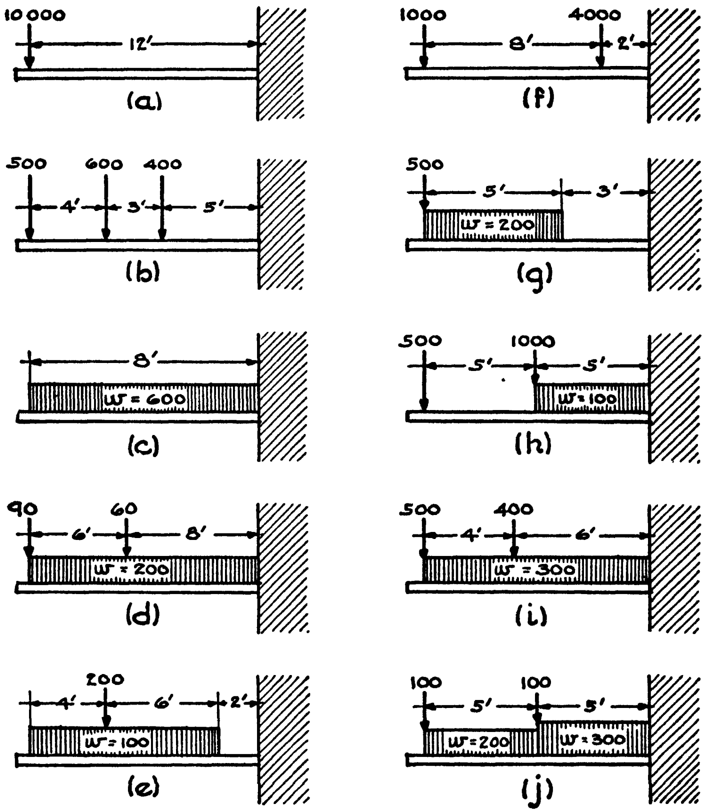


FIG. 33

SOLUTION. Computing the shear,

$$V_{(x=1)} = -2{,}000\#$$

$$V_{(x=8)} = -2{,}000\#$$

$$V_{(x=10)} = -[2{,}000 + (600 \times 2)] = -3{,}200\#$$

$$V_{(x=14)} = -[2{,}000 + (600 \times 6)] = -5{,}600\#$$

Computing the moments,

$$M_{(x=0)} = 0$$

$$M_{(x=4)} = -(2{,}000 \times 4) = -8{,}000'\#$$

$$M_{(x=8)} = -(2{,}000 \times 8) = -16{,}000'\#$$

$$M_{(x=10)} = -[(2{,}000 \times 10) + (600 \times 2 \times 1)] = -21{,}200'\#$$

$$M_{(x=14)} = -[(2{,}000 \times 14) + (600 \times 6 \times 3)] = -38{,}800'\#$$

The maximum shear is 5,600#, and the maximum bending moment is 38,800′#. In cantilever beams the maximum values for both shear and bending moment are found to be at the support, and the beams are concave downward.

PROBLEMS

33-A-B-C-D-E-F-G-H-I-J. Draw the shear and bending-moment diagrams for the beams shown in Fig. 33. What are the maximum shear and the maximum bending moment in each beam?

34. **Bending-Moment Formulas.** The method of computing bending moments previously given in this chapter enables us to find the maximum value for almost any condition of beam and loading that is encountered in practice. However, certain conditions occur so frequently that the work may be shortened by using formulas which give directly the maximum bending moment. All handbooks give such formulas, but only two of the most commonly used will be derived here.

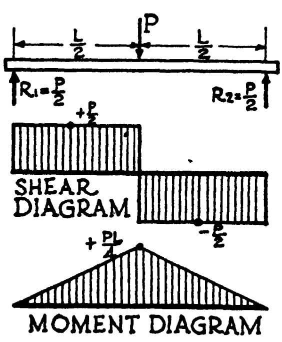


Fig. 34

35. **Concentrated Load at Center of Span.** A simple beam having a concentrated load at the center of the span occurs very frequently in practice. Call the load P and the span between supports L. Then each reaction will be $\frac{P}{2}$, and the load will occur at $\frac{L}{2}$ distance from R_1 (Fig. 34). When the shear diagram is drawn, it is seen that the maximum bending moment occurs at the center of

the span. Consequently, let us write the value of the bending moment at this section.

$$M_{\left(x=\frac{L}{2}\right)} = \frac{P}{2} \times \frac{L}{2}$$

or

$$M = \frac{PL}{4}$$

This formula is well worth remembering. Observe how quickly problems are solved by its use.

EXAMPLE

EXAMPLE. A simple beam 20′ in length has a concentrated load of 8,000# at the center of the span. Compute the maximum bending moment.

SOLUTION. The value of the maximum bending moment for this condition is $M = \frac{PL}{4}$. Therefore

$$M = \frac{8{,}000 \times 20}{4} = 40{,}000'\#$$

If we wish the result in inch-pounds, *as we usually do*, we simply multiply by 12. That is,

$$40{,}000'\# = (40{,}000 \times 12)$$

or

$$480{,}000''\#$$

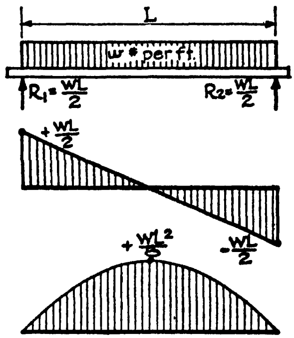


FIG. 35

36. Simple Beam with Uniformly Distributed Load. This is probably the most common beam problem. It occurs time and time again. Let us call the span L and the load w pounds per linear foot (Fig. 35). The total load will be wL, hence each reaction will be $\frac{wL}{2}$. The maximum bending moment occurs at the center of the span, at $\frac{L}{2}$ from R_1. Writing the value of M for this section, we have

$$M_{\left(x=\frac{L}{2}\right)} = \left(\frac{wL}{2} \times \frac{L}{2}\right) - \left(\frac{wL}{2} \times \frac{L}{4}\right)$$

$$M = \frac{wL^2}{4} - \frac{wL^2}{8}$$

$$M = \frac{wL^2}{8}$$

If, instead of being given the load per linear foot, we have the total uniformly distributed load, we call the total load W. Since $wL = W$, the value of the maximum bending moment can be written $M = \frac{wL^2}{8}$ or $\frac{WL}{8}$. Remember this equation. You will use it many times. See how convenient it is in a practical example.

EXAMPLE

EXAMPLE. A simple beam 14′ in length has a uniformly distributed load of 800 pounds per linear foot. Compute the maximum bending moment.

SOLUTION. The formula for the maximum bending moment for a simple beam having a uniformly distributed load is $M = \frac{wL^2}{8}$. Substituting these values,

$$M = \frac{800 \times 14 \times 14}{8}$$

$$M = 19{,}600'\#$$

$$M = 19{,}600 \times 12 = 235{,}200''\#$$

Suppose in this problem we had been given the total load, 11,200# instead of 800#/ft. Then

$$M = \frac{WL}{8}$$

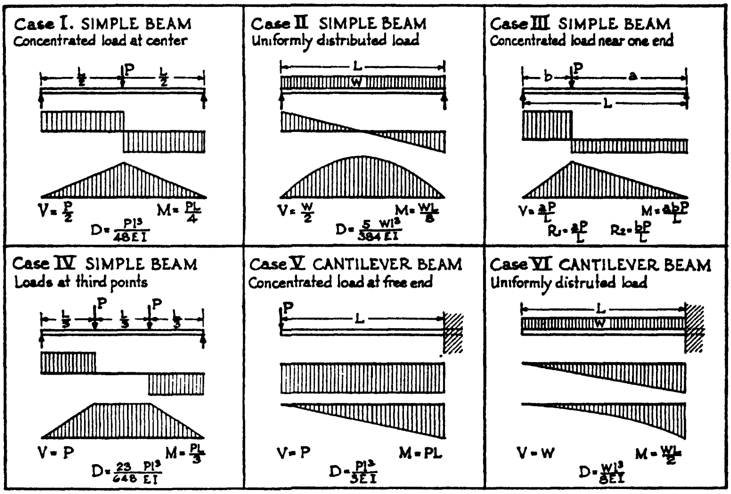
$$M = \frac{11{,}200 \times 14}{8} = 19{,}600'\#$$

$$19{,}600'\# = 235{,}200''\#$$

The result, of course, is the same.

37. Typical Beams. Shear and Moment Formulas. Some of the most typical beams and loads are shown in Table 4. The values of maximum shear, bending moments, and deflection are given. In general, L = the span in feet and l = the span in inches.

Table 4



If the loads are given in pounds and the span in feet, the bending moment will be in units of foot-pounds. Particular attention is called to l in the formulas for maximum deflection. In this table the formulas for deflection all give the length in terms of l, that is, inches.

THEORY OF BENDING AND PROPERTIES OF SECTIONS

38. Theory of Bending. Figure 36 (a) shows a simple beam having one concentrated load P. Figure 36 (b) shows an enlarged portion of the left-hand end up to the section X–X. From the previous discussions we know that the reaction R_1 tends to cause a clockwise rotation about the point A in the section under consideration; this we know to be the bending moment. In this type of beam, the fibers in the upper part of the beam are in compression and those in the lower part are in tension. There is an imaginary

plane separating the compressive and tensile stresses; it is called the neutral surface or neutral axis, and at this plane there are neither compressive nor tensile stresses with respect to bending.

Call the sum of all the compressive stresses C, and the sum of all the tensile stresses T. Now it is the sum of the moments of the stresses in the section that holds the beam in equilibrium; this is called the resisting moment and is equal to the bending moment in magnitude. The bending moment about the point A is $R_1 \times x$, and the resisting moment about the same point is $(C \times y) +$

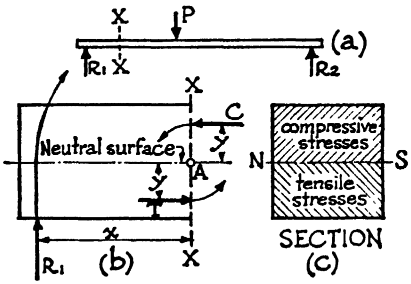


FIG. 36

$(T \times y)$. The bending moment tends to cause a clockwise rotation, and the resisting moment tends to cause a counterclockwise rotation. If the beam is in equilibrium these moments are equal, or

$$R_1 \times x = (C \times y) + (T \times y)$$

This is the theory of bending in beams. For any type of beam we can compute the bending moment, and if we wish to select (design) a beam to withstand this tendency to bend, we must select a member having a cross section of such shape, area, and material that the resisting moment will have a magnitude equal to the bending moment.

39. Flexure Formula. The flexure formula, $\frac{M}{f} = S$, is used in the design of beams. In it, M = the maximum bending moment in inch-pounds, f = the allowable unit stress on the fiber most remote from the neutral surface in pounds per square inch, and S = the section modulus of the cross section of the member in inches^3. For those who wish to know the principles upon which this formula is based, the following brief derivation is given. You

will never need to derive this formula, but you will be called upon to use it many, many times. Notice how readily it is applied in the example given.

Figure 37 represents the end of any beam subject to bending stresses; it may or may not be symmetrical about the neutral surface. If the greatest fiber stress in the section does not exceed the elastic limit of the material, the stresses are directly proportional to their distances from the neutral surface. That is to say, if one fiber is twice the distance from the neutral surface that another is, the fiber farther away will have twice the stress of the

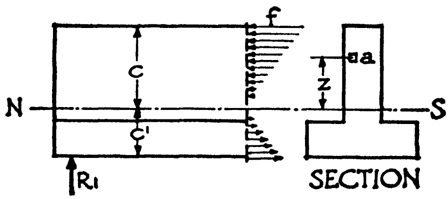


FIG. 37

other. These stresses are indicated in the figure by the small lines with arrows, representing the compressive and tensile stresses.

Call c the distance of the fiber farthest from the neutral surface, and f the unit stress on this fiber. If c is in inches, the unit stress on a fiber at 1″ distance will be $\frac{f}{c}$. Regardless of the denomination, the unit stress on a fiber at unity distance is $\frac{f}{c}$.

Now imagine an infinitely small area a at z distance from the neutral surface. The unit stress on this fiber will be $\frac{f}{c} \times z$, and since this small area contains a square inches, the total stress on fiber a will be $\frac{f}{c} \times z \times a$, and the *moment* of the stress on fiber a at z distance will be $\frac{f}{c} \times z \times a \times z$, or $\frac{f}{c} \times a \times z^2$. We know, however, that there is an infinite number of these minute areas, and if we use the symbol Σ to represent the sum of an infinite number, we

can write $\Sigma \frac{f}{c} \times a \times z^2$, which really means the sum of the moments of all the stresses in the cross section with respect to the neutral surface. This we know is the *resisting moment* and is equal to the bending moment.

Therefore

$$M = \frac{f}{c} \Sigma a \times z^2$$

The term $\Sigma a \times z^2$ may be read "the sum of the products of all the elementary areas by the square of their distances from the neutral surface." We call this the *moment of inertia*, and represent it by the letter I. Therefore, substituting in the above, $M = \frac{f}{c} I$ or

$$\frac{M}{f} = \frac{I}{c}$$

This is known as the *flexure formula*, and by its use we may design any beam which is composed of one material. The quantity $\frac{I}{c}$ is known as the *section modulus;* it is represented by the letter S. Therefore

$$\frac{M}{f} = S$$

The application of the flexure formula, $\frac{M}{f} = S$, is surprisingly simple. If we wish to design a beam we compute the maximum bending moment and divide it by the allowable bending unit stress of the material. The quotient gives us the required section modulus S, and we merely look in the manufacturers' tables and select a beam having an S of the same or greater magnitude. What could be simpler?

EXAMPLE

EXAMPLE. What steel I-beam may be used to support a uniformly distributed load of 24,000# if the span of the beam is 12′?

SOLUTION. The maximum bending moment for this beam is $\frac{Wl}{8}$ (Art. 36), or

$$M = \frac{24{,}000 \times 12}{8} = 36{,}000'\# \qquad 36{,}000 \times 12 = 432{,}000''\#$$

The flexure formula is $\frac{M}{f} = S$, and $f = 18{,}000\#/\text{in}^2$ (Table 2). Therefore $\frac{432{,}000}{18{,}000} = 24\text{in}^3$ = the section modulus required. By referring to Table 5, we find that a 10″ I 25.4# has a section modulus of 24.4in^3; therefore this beam may be used.

A simpler method of finding the proper I-beam for uniformly distributed loads is to use Tables 23-A and 23-B, as explained in Art. 64. This table, however, applies only to uniformly distributed loads, but $\frac{M}{f} = S$, the flexure formula, is general and is applicable to any type of beam and loading.

40. **Moment of Inertia.** In developing the flexure formula we found the term $\Sigma a \times z^2$. This is the *moment of inertia*, which may be defined as *the sum of the products obtained by multiplying all the infinitely small areas by the square of their distances to the neutral axis.* It is represented by I. The elementary areas are in units of square inches, and square inches multiplied by a distance squared gives inches to the fourth power. For instance, 24in^4 is read "24 inches to the fourth power."

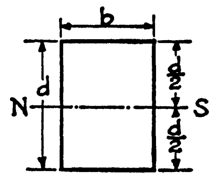


FIG. 38

In Fig. 38, we have a rectangular cross section of breadth b and depth d. The neutral surface passes through the centroid of the cross section; it is represented by the line NS and is $\frac{d}{2}$ distance from the upper and lower edges. It can be shown that *the moment of inertia of a rectangular cross section about an axis passing through its centroid, parallel to the base, is* $\frac{bd^3}{12}$. By the use of this formula, $I = \frac{bd^3}{12}$, the value of the moment of inertia for rectangular cross sections is quickly found. The values of I for standard timber sizes are given in Table 12, and are computed for actual or dressed sizes.

EXAMPLES

EXAMPLE. Compute the value of the moment of inertia of a 6″ x 12″ timber section. (See Table 12.)

SOLUTION. The 6″ x 12″ is the "nominal size," and the "American standard dressed size" (which is the actual size) is 5.5″ x 11.5″. Then

$$I = \frac{bd^3}{12} = \frac{5.5 \times 11.5 \times 11.5 \times 11.5}{12} = 697.06\text{in}^4$$

The moment of inertia of a rectangular cross section about an axis through its base is $\frac{bd^3}{3}$. This value is often convenient in computing I for unsymmetrical sections, such as an angle section.

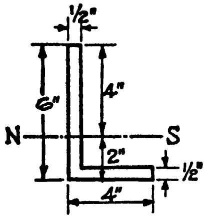


FIG. 39

EXAMPLE. Compute the moment of inertia of a 6″ x 4″ x ½″ angle, assuming the neutral axis to be 2″ from the base of the short arm (Fig. 39).

SOLUTION. First make a sketch showing the dimensions. Taking the portion of the long arm above the neutral axis and writing the value of I about the base of the rectangle,

$$I = \frac{bd^3}{3} = \frac{0.5 \times 4 \times 4 \times 4}{3} = 10.66\text{in}^4$$

In the same manner, the value of I for the rectangle below the NS, having a breadth 4″ and depth 2″ is

$$I = \frac{bd^3}{3} = \frac{4 \times 2 \times 2 \times 2}{3} = 10.66\text{in}^4$$

From this last quantity, we must subtract I for the rectangle of width 3.5″ and depth 1.5″ in which no material occurs and which was included in the previous equation. Therefore

$$I = \frac{bd^3}{3} = \frac{3.5 \times 1.5 \times 1.5 \times 1.5}{3} = 3.93\text{in}^4$$

Then $10.66 - 3.93 = 6.73\text{in}^4$, the moment of inertia of the area of the angle section below the NS. This quantity, added to the I for the section above the NS $= 10.66 + 6.73 = 17.39\text{in}^4$, the value of I for the entire section taken about the neutral surface.

The previous example illustrates one method of finding the moment of inertia of an unsymmetrical section, but it is not necessary to compute I for the commonly used sections since they are found in the manufacturers' tables. Moments of inertia for steel sections are found in Tables 5 to 11 inclusive.

PROBLEMS

40-A-B-C-D-E-F. Compute the moment of inertia about the neutral surface of the beam sections shown in Fig. 40.

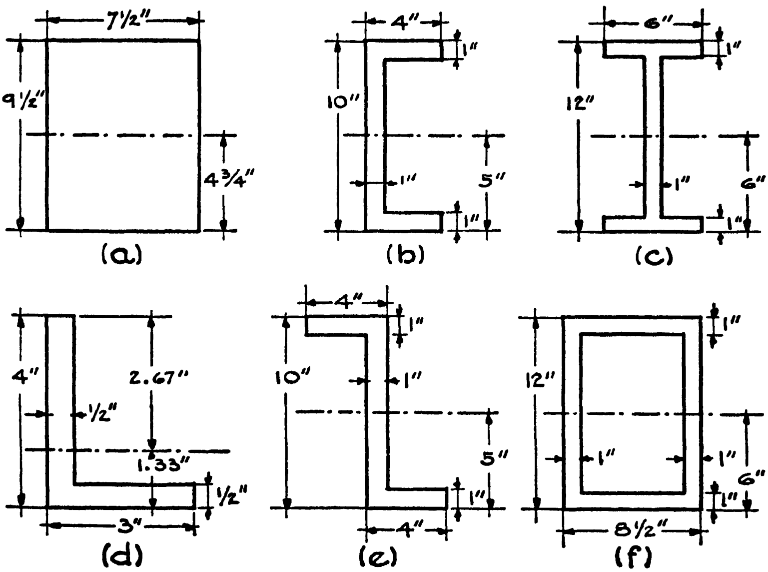


FIG. 40

41. Centroids. Center of Gravity. *The centroid of a plane surface is a point which corresponds to the center of gravity of a very thin homogeneous plate of the same area and shape.* It can be shown that the neutral surface of a beam passes through the centroid of the section. For symmetrical sections such as rectangles and I-sections

it is obvious that the centroid is located at a point ½ the distance between the upper and lower surfaces of the section. In the flexure formula, $\frac{M}{f} = \frac{I}{c}$, the letter c is used to represent the distance of the fiber most remote from the neutral surface. Therefore c for rectangles is equal to ½ the depth, or $\frac{d}{2}$. For a 12″ I-beam, $c = \frac{12}{2} = 6''$. It is important that we know the position of the centroid in order that we may determine the distance c. For unsymmetrical sections the position of the centroid must be computed.

The statical moment of a plane area with respect to an axis is the product of the area times the normal distance of the centroid of the area to the axis. *If an area is divided into a number of parts, the statical moment of the entire area is equal to the sum of the statical moments of the parts.* This is our key for locating the centroid. The manufacturers of structural steel have published data giving the position of the neutral surface, so that it is seldom necessary to compute it. Tables 10 and 11, for instance, give the values x and y, the distance of the neutral surface from the back of the legs. The following example shows how the distance is computed.

EXAMPLE

Example. Figure 41 is a beam section unsymmetrical with respect to the horizontal axis. Find the value of c, the distance of the neutral surface from the fiber most remote.

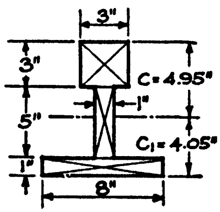


Fig. 41

Solution. It is not always possible to tell by observation whether the centroid is nearer the top or bottom surface of the area. In this instance let us write an equation of moments about an axis through the *uppermost* surface. First divide the section into any number of convenient areas as shown by the diagonals, in this case three areas. The area of the upper flange is $9 in^2$, and its centroid is 1½″ from the assumed axis; the web has an area of $5 in^2$, and its centroid is 5½″ from the same axis. Likewise, the bottom flange contains $8 in^2$, and 8½″ is the distance of its centroid

to the axis. The entire area is 22in^2, and its centroid is c inches from the axis. Then, since the sum of the moments of the parts is equal to the moment of the whole area, we may write

$$(9 \times 1\tfrac{1}{2}) + (5 \times 5\tfrac{1}{2}) + (8 \times 8\tfrac{1}{2}) = 22 \times c$$

$$109 = 22 \times c$$

$$c = 4.95''$$

The distance of the centroid to the lowest surface is

$$9 - 4.95 = 4.05''$$

Call this distance c_1. This last value may be checked by writing an equation of moments about an axis taken through the *bottom* surface; thus

$$(8 \times 0.5) + (5 \times 3.5) + (9 \times 7.5) = 22 \times c_1$$

$$c_1 = 4.05''$$

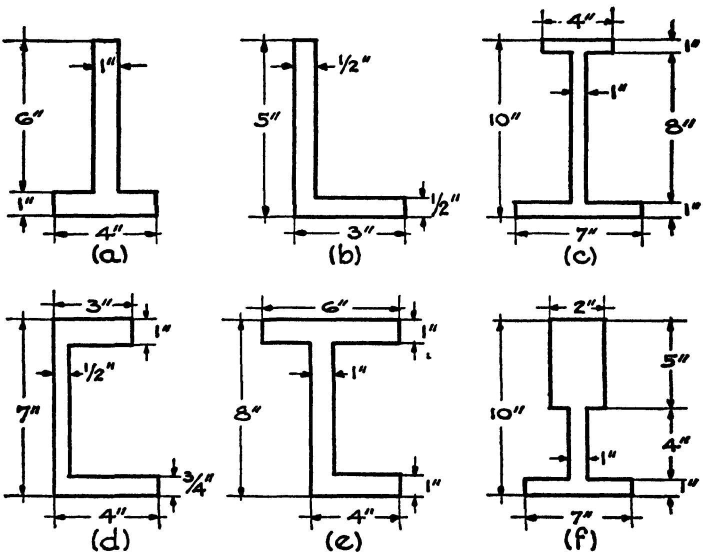


FIG. 42

PROBLEMS

41-A-B-C-D-E-F. Compute the value of c for the sections shown in Fig. 42

42. Transferring Moments of Inertia. Referring to Fig. 43, let I be the moment of inertia of the angle section, having an area A, about an axis X–X which passes through the centroid of the section. Let I_1 be the moment of inertia of the same section with respect to the axis Y–Y which is parallel to X–X at h distance. It can be shown that

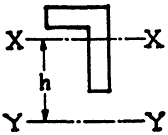


FIG. 43

$$I_1 = I + Ah^2$$

which may be stated thus: *The moment of inertia of a plane surface with respect to any axis is equal to the moment of inertia with respect to a parallel axis through its centroid plus the area of the surface multiplied by the square of the distance between the axes.*

To illustrate the use of the formula, we know the value of I for a rectangle of breadth b and depth d, about an axis through its centroid parallel to the base, to be $\frac{bd^3}{12}$. Now if we wish to know the moment of inertia about an axis taken through the base, we can use the above formula, the distance between the axes being $\frac{d}{2}$. Then

$$I_1 = I + Ah^2$$

$$I_1 = \frac{bd^3}{12} + \left[bd \times \left(\frac{d}{2}\right)^2\right]$$

$$I_1 = \frac{bd^3}{12} + \frac{bd^3}{4} = \frac{bd^3}{3}$$

which is the value given in Art. 40.

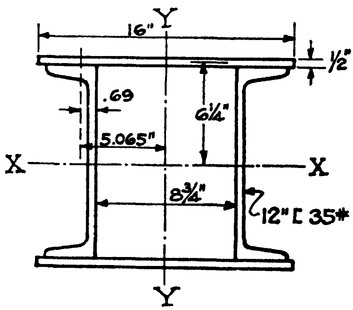


FIG. 44

This equation for transferring moments of inertia from one axis to another is used in computing I for built-up sections such as plate girders and columns. For the sections most commonly used in practice the various properties, including the moment of inertia, may be found in the manufacturers' tables. The following example shows how the formula is applied.

EXAMPLE

Example. Compute the moment of inertia of a built-up column section composed of two 12″ 35# channels and two 16″ x ½″ plates, about the X–X axis (Fig. 44).

Solution. By referring to Table 9, we find I for a 12″ 35# [about the X–X axis to be 178.8in^4. Since there are two channels,

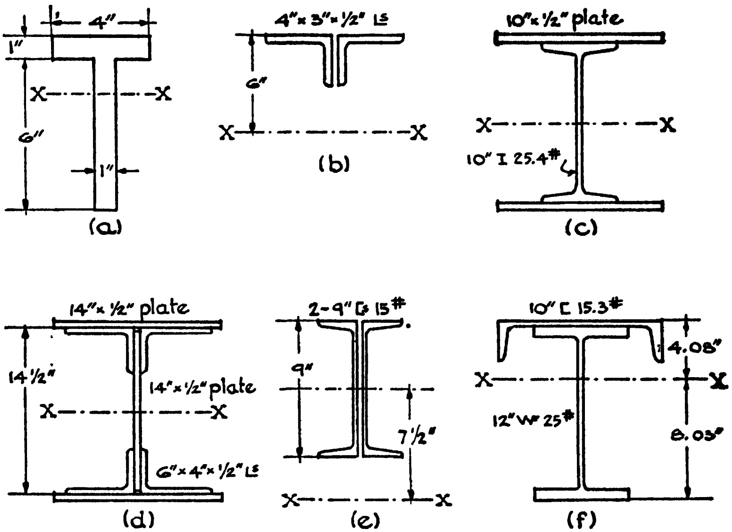


Fig. 45

$2 \times 178.8 = 357.6\text{in}^4$, the moment of inertia of both channels. I for a 16″ x ½″ plate about an axis through its centroid is $\frac{bd^3}{12}$,

or $I = \frac{16 \times 0.5 \times 0.5 \times 0.5}{12} = 0.166\text{in}^4$. The distance between the centroid of the plate and the axis X–X is 6.25″, and the area of one plate is 8in^2; therefore, substituting in the formula $I_1 = I + Ah^2$, $I_1 = 0.166 + (8 \times 6.25^2)$, or $I_1 = 312.67\text{in}^4$, which is the value of I for one plate about the X–X axis. I for two plates will be $2 \times 312.67 = 625.34\text{in}^4$. Adding the moments of inertia of the channels and plates we have $357.6 + 625.34 = 982.94\text{in}^4$, the value of I for the entire cross section about the X–X axis. For the

following problems refer to tables giving properties of sections and make use of any information which is applicable.

PROBLEMS

42-A-B-C-D-E-F. Compute I for the beam sections shown in Fig. 45 with respect to the X–X axes.

43. **Section Modulus.** In Art. 39 we found the flexure formula to be $\frac{M}{f} = \frac{I}{c}$. The term $\frac{I}{c}$ is called the *section modulus* and is represented by the letter S; *it is the quotient obtained by dividing the moment of inertia by the distance of the fiber most remote from the neutral surface.* Since I is in units of inches to the fourth power and c is in linear inches, $\frac{I}{c}$ or S is in inches to the third power, as 230in^3.

Referring to Fig. 38, a rectangular cross section of breadth b and depth d, we know that the moment of inertia about the X–X axis is $\frac{bd^3}{12}$ and that $c = \frac{d}{2}$. Therefore $\frac{I}{c}$ or $S = \frac{bd^3}{12} \div \frac{d}{2} = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6}$. It is often convenient to use this value directly. It applies, of course, only to rectangular cross sections.

EXAMPLES

Example. Compute the section modulus of a 6″ x 12″ timber beam.

Solution. The nominal size of the beam is 6″ x 12″ and the actual size is 5½″ x 11½″. Since $S = \frac{bd^2}{6}$, $S = \frac{5.5 \times 11.5 \times 11.5}{6}$ = 121.23in^3. (See Table 12.)

Example. Compute the section modulus of a 4″ x 3″ x ½″ angle with respect to an axis through the centroid parallel to the short leg.

Solution. By the methods explained in Arts. 40 and 41, c is found to be 2.67″ and I = 5.1in^4. Since $S = \frac{I}{c}$, $S = \frac{5.1}{2\,67}$ = 1.9in^3.

PROBLEMS

43-A. Compute the section modulus of a 6″ x 8″ timber beam.

43-B. Table 5 shows the S for a 10″ I 25.4#, to be 24.4in^3. Show why this is true.

43-C. Figure 42 (a) indicates a T-beam section. Compute the section modulus.

43-D. A 20″ I 70# has 10″ x ⅝″ cover plates at each flange. Compute the section modulus.

43-E. Compute the section modulus of a 14″ WF 43# section with respect to the Y–Y axis and check the result with Table 6.

43-F. Compute the section modulus of the built-up section shown in Fig. 45(f).

43-G. Compute the section modulus of a 10″ x 12″ timber beam.

44. **Radius of Gyration.** In Art. 39 we found the moment of inertia to be a name given to the quantity $\Sigma a \times z^2$, that is, $I = \Sigma a \times z^2$. Now Σa is actually the sum of all the infinitely small areas which of course is A, and z is a variable having values between 0 and c. If we imagine a point where the whole area might be concentrated, and if the area multiplied by the square of the distance from this point to the neutral surface produces a quantity equal to the moment of inertia of the section, the point is called the *center of gyration*, and the distance from the point to the neutral surface is the *radius of gyration*. That is,

$$I = \Sigma a \times z^2 \qquad \text{or} \qquad I = Ar^2 \qquad \text{or} \qquad r = \sqrt{\frac{I}{A}}$$

in which r = the radius of gyration, in inches

I = the moment of inertia of the cross section, in inches to the fourth power

A = area of the cross section, in square inches.

The radius of gyration is a property of cross sections which depends on their size and shape. It is most frequently used in the design of columns. If a section is symmetrical about both major axes, the moment of inertia and consequently the radius of gyration are the same for each axis. But most column sections, particularly steel columns, are not symmetrical about the two major axes, and *in the design of columns the least moment of inertia and, therefore, the least radius of gyration is the one used in computa-*

tions. Note in Tables 10 and 11 that the least radius of gyration of angle sections occurs about the Z–Z axis.

EXAMPLE

EXAMPLE. Compute the least radius of gyration of the column section shown in Fig. 44.

SOLUTION. First consider the section with respect to the X–X axis. In the example under Art. 42 we found I to be 982.94in^4. The section is made up of two plates and two channels. The area of the plates is $2 \times 16 \times 0.5 = 16$in^2. Table 9 shows that the area of one 12″ 35# [is 10.26in^2. Two channels = $10.26 \times 2 = 20.52$in^2. Therefore $16 + 20.52 = 36.52$in^2, the area of the entire section.

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{982.94}{36.52}} = 5.19''$$

the radius of gyration with respect to the X–X axis.

We must now consider the Y–Y axis. I for one plate $= \frac{bd^3}{12} = \frac{0.5 \times 16 \times 16 \times 16}{12} = 170.6$in^4. For two plates $I = 170.6 \times 2 = 341.2$in^4. Referring to Table 9, we find I for one channel about an axis through its centroid parallel to the web to be 5.9in^4, and the distance of this axis to the back of the web is 0.69″. The distance for transferring moments of inertia $h = 0.69 + \frac{8.75}{2} = 5.065''$. The area of one channel is 10.26in^2. The moment of inertia about the Y–Y axis is found by use of the formula $I_1 = I + Ah^2$, or $I_1 = 5.9 + (10.26 \times 5.065^2) = 269.11$in^4. I for two channels is $269.11 \times 2 = 538.22$in^4. Then I for the entire section about the Y–Y axis is $538.22 + 341.2 = 879.42$in^4.

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{879.42}{36.52}} = 4.9''$$

the radius of gyration with respect to the Y–Y axis.

Since 4.9″ is less than 5.19″, 4.9″ is the least radius of gyration of the section.

PROBLEMS

44-A. A 10″ I 30# is to be used as a column. Compute the least radius of gyration. (See Table 5.)

44-B. Compute the least radius of gyration of a 12″ WF 65#. (See Table 6.)

44-C. Compute the least radius of gyration of the built-up section shown in Fig. 45 (*c*).

44-D. Compute the radius of gyration, about the $Y-Y$ axis, of the column section shown in Fig. 45 (*d*).

44-E. Compute the least radius of gyration of a 16″ WF 36#.

44-F. Compute the least radius of gyration of the built-up section shown in Fig. 45 (*f*).

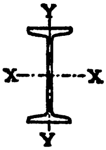
44-G. Compute the least radius of gyration of the T-section shown in Fig. 45 (*a*).

45. **Properties or Elements of Structural Shapes.** Handbooks published by the manufacturers contain much valuable information concerning general engineering subjects and also specific data relating to properties or elements of the structural shapes rolled by their mills. Tables 5, 6, 7, 8, 9, 10, 11, 23-A, 23-B, 26, 27, 28, and 29 have been compiled with permission from handbooks issued by The American Institute of Steel Construction, The Carnegie Steel Company, and The Bethlehem Steel Company. The tables are necessarily brief and are given here as a means of ready reference. It is well to note that most structural shapes have two major axes, designated as $X-X$ and $Y-Y$, the position in which the member is placed determining the axis to be considered. I-beams, for example, are invariably used with the web vertical, and hence the $X-X$ or horizontal axis determines the section modulus. Generally the lightest weight section of a given depth is the most economical selection.

In designing columns remember that the least radius of gyration is the one to consider unless the framing determines the direction in which the member tends to bend.

Table 12 gives the properties of American Standard Yard Lumber and Timber Sizes. Notice that both the nominal and dressed sizes are given and that the properties are based on actual or dressed sizes.

Table 5. Properties of American Standard I-Beams



Nominal Size	Weight per Foot	Area of Section	Depth of Section	Width of Flange	Web Thickness	Axis X–X			Axis Y–Y		
						I	*S*	*r*	*I*	*S*	*r*
In	Lb	In^2	In	In	In	In^4	In^3	In	In^4	In^3	In
24 x 7⅞	120.0	35.13	24.00	8.048	.798	3,010.8	250.9	9.26	84.9	21.1	1.56
	115.0	33.67	24.00	7.987	.737	2,940.5	245.0	9.35	82.8	20.7	1.57
	110.0	32.18	24.00	7.925	.675	2,869.1	239.1	9.44	80.6	20.3	1.58
	105.9	30.98	24.00	7.875	.625	2,811.5	234.3	9.53	78.9	20.0	1.60
24 x 7	95.0	27.79	24.00	7.186	.686	2,301.5	191.8	9.08	47.0	13.0	1.30
	85.0	24.84	24.00	7.063	.563	2,159.8	180.0	9.33	44.2	12.5	1.33
	79.9	23.33	24.00	7.000	.500	2,087.2	173.9	9.46	42.9	12.2	1.36
20 x 7	100.0	29.20	20.00	7.273	.873	1,648.3	164.8	7.51	52.4	14.4	1.34
	95.0	27.74	20.00	7.200	.800	1,599.7	160.0	7.59	50.5	14.0	1.35
	90.0	26.26	20.00	7.126	.726	1,550.3	155.0	7.68	48.7	13.7	1.36
	85.0	24.80	20.00	7.053	.653	1,501.7	150.2	7.78	47.0	13.3	1.38
	81.4	23.74	20.00	7.000	.600	1,466.3	146.6	7.86	45.8	13.1	1.39
20 x 6¼	70.0	20.42	20.00	6.317	.567	1,214.2	121.4	7.71	28.9	9.2	1.19
18 x 6	70.0	20.46	18.00	6.251	.711	917.5	101.9	6.70	24.5	7.8	1.09
	65.0	18.98	18.00	6.169	.629	877.7	97.5	6.80	23.4	7.6	1.11
	60.0	17.50	18.00	6.087	.547	837.8	93.1	6.92	22.3	7.3	1.13
	54.7	15.94	18.00	6.000	.460	795.5	88.4	7.07	21.2	7.1	1.15
15 x 6	75.0	21.85	15.00	6.278	.868	687.2	91.6	5.61	30.6	9.8	1.18
	70.0	20.38	15.00	6.180	.770	659.6	87.9	5.69	28.8	9.3	1.19
	65.0	18.91	15.00	6.082	.672	632.1	84.3	5.78	27.2	8.9	1.20
	60.8	17.68	15.00	6.000	.590	609.0	81.2	5.87	26.0	8.7	1.21
15 x 5½	55.0	16.06	15.00	5.738	.648	508.7	67.8	5.63	17.0	5.9	1.03
	50.0	14.59	15.00	5.640	.550	481.1	64.2	5.74	16.0	5.7	1.05
	45.0	13.12	15.00	5.542	.452	453.6	60.5	5.88	15.0	5.4	1.07
	42.9	12.49	15.00	5.500	.410	441.8	58.9	5.95	14.6	5.3	1.08
12 x 5¼	55.0	16.04	12.00	5.600	.810	319.3	53.2	4.46	17.3	6.2	1.04
	50.0	14.57	12.00	5.477	.687	301.6	50.3	4.55	16.0	5.8	1.05
	45.0	13.10	12.00	5.355	.565	284.1	47.3	4.66	14.8	5.5	1.06
	40.8	11.84	12.00	5.250	.460	268.9	44.8	4.77	13.8	5.3	1.08
12 x 5	35.0	10.20	12.00	5.078	.428	227.0	37.8	4.72	10.0	3.9	.99
	31.8	9.26	12.00	5.000	.350	215.8	36.0	4.83	9.5	3.8	1.01
10 x 4¾	40.0	11.69	10.00	5.091	.741	158.0	31.6	3.68	9.4	3.7	.90
	35.0	10.22	10.00	4.944	.594	145.8	29.2	3.78	8.5	3.4	.91
	30.0	8.75	10.00	4.797	.447	133.5	26.7	3.91	7.6	3.2	.93
	25.4	7.38	10.00	4.660	.310	122.1	24.4	4.07	6.9	3.0	.97

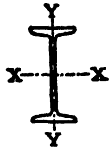
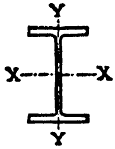


Table 5. Properties of American Standard I-Beams (Continued)

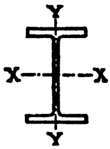
Nominal Size	Weight per Foot	Area of Section	Depth of Section	Width of Flange	Web Thickness	Axis X–X			Axis Y–Y		
						I	*S*	*r*	*I*	*S*	*r*
In	Lb	In^2	In	In	In	In^4	In^3	In	In^4	In^3	In
8 x 4	25.5	7.43	8.00	4.262	.532	68.1	17.0	3.03	4.7	2.2	.80
	23.0	6.71	8.00	4.171	.441	64.2	16.0	3.09	4.4	2.1	.81
	20.5	5.97	8.00	4.079	.349	60.2	15.1	3.18	4.0	2.0	.82
	18.4	5.34	8.00	4.000	.270	56.9	14.2	3.26	3.8	1.9	.84
7 x 3¾	20.0	5.83	7.00	3.860	.450	41.9	12.0	2.68	3.1	1.6	.74
	17.5	5.09	7.00	3.755	.345	38.9	11.1	2.77	2.9	1.6	.76
	15.3	4.43	7.00	3.660	.250	36.2	10.4	2.86	2.7	1.5	.78
6 x 3⅜	17.25	5.02	6.00	3.565	.465	26.0	8.7	2.28	2.3	1.3	.68
	14.75	4.29	6.00	3.443	.343	23.8	7.9	2.36	2.1	1.2	.69
	12.5	3.61	6.00	3.330	.230	21.8	7.3	2.46	1.8	1.1	.72
5 x 3	14.75	4.29	5.00	3.284	.494	15.0	6.0	1.87	1.7	1.0	.63
	12.25	3.56	5.00	3.137	.347	13.5	5.4	1.95	1.4	.91	.63
	10.0	2.87	5.00	3.000	.210	12.1	4.8	2.05	1.2	.82	.65

Table 6. Properties of WF Sections



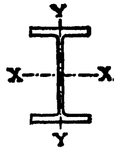
Nominal Size	Weight per Foot	Area of Section	Depth of Section	Flange Width	Flange Thickness	Web Thickness	Axis X-X I	Axis X-X S	Axis X-X r	Axis Y-Y I	Axis Y-Y S	Axis Y-Y r
In	Lb	In²	In	In	In	In	In⁴	In³	In	In⁴	In³	In
36 x 16½	300	88.17	36.72	16.655	1.680	.945	20,290 2	1,105 1	15.17	1,225.2	147.1	3.73
	230	67.73	35.88	16.475	1.260	.765	14,988.4	835.5	14.88	870.9	105.7	3.59
36 x 12	160	47.09	36.00	12.000	1.020	.653	9,738.8	541.0	14.38	275.4	45.9	2.42
	150	44.16	35.84	11.972	.940	.625	9,012.1	502.9	14.29	250.4	41.8	2.38
33 x 15¾	220	64.73	33.25	15.810	1.275	.775	12,312.1	740.6	13.79	782.4	99.0	3.48
	200	58.79	33.00	15.750	1.150	.715	11,048.2	669.6	13.71	691.7	87.8	3.43
33 x 11½	132	38.84	33.15	11.510	.880	.580	6,856.8	413.7	13.29	207.8	36.1	2.31
	125	36.78	33.00	11.500	.805	.570	6,354.7	385.1	13.14	188.2	32 7	2.26
30 x 15	190	55.90	30.12	15.040	1.185	.710	8,825.9	586.1	12.57	624.6	83.1	3.34
	172	50.65	29.88	14.985	1.065	.655	7,891.5	528.2	12.48	550.1	73.4	3.30
30 x 10½	116	34.13	30 00	10.500	.850	.564	4,919.1	327 9	12.00	153.2	29.2	2.12
	108	31.77	29.82	10.484	.760	.548	4,461.0	299.2	11.85	135.1	25.8	2.06
27 x 14	163	47.93	27.12	14 035	1.095	.670	6,141.5	452.9	11.32	468.7	66.8	3.13
	145	42.68	26.88	13.965	.975	.600	5,414.3	402.9	11.26	406.9	58.3	3.09
27 x 10	98	28.82	27 00	10.000	.792	.500	3,446.5	255.3	10.94	122.9	24.6	2.07
	91	26.77	26.84	9.983	.712	.483	3,129.2	233.2	10.81	109.0	21.8	2.02
24 x 14	150	44.10	24.56	14.063	1.055	.628	4.733.5	385.5	10.36	452.5	64.3	3.20
	130	38.21	24.25	14.000	.900	.565	4,609.5	330.7	10.24	375.2	53.6	3.13
24 x 12	120	35.29	24.31	12.088	.930	.556	3,635.3	299.1	10.15	254.0	42.0	2.68
	100	29.43	24.00	12.000	.775	.468	2,987.3	248.9	10.08	203.5	33.9	2.63
24 x 9	80	23.54	24.00	9.000	.727	.455	2,229.7	195 8	9.73	82.4	18 3	1.87
	74	21.77	23.87	8.975	.662	.430	2,033.8	170.4	9.67	73.8	16.5	1.84
21 x 13	132	38.81	21.31	13.087	1.020	.614	3,141.6	294.8	9.00	353.8	54.1	3.02
	112	32.93	21.00	13.000	.865	.527	2,620.6	249.6	8.92	289.7	44.6	2.96
21 x 9	96	28.21	21.14	9.038	.935	.575	2,088.9	197.6	8.60	109.3	24.2	1.97
	82	24.10	20.86	8.962	.795	.499	1,752.4	168.0	8.53	89.6	20.0	1.93
21 x 8¼	63	18.52	21.00	8.250	.620	.410	1,343.6	128.0	8.52	53.8	13.0	1.70
	59	17.36	20.91	8.230	.575	.390	1,246.8	119.3	8.47	49.2	12.0	1.68
18 x 11¾	105	30.86	18.32	11.792	.911	.554	1,852.5	202.2	7.75	231.0	39.2	2.73
	96	28.22	18.16	11.750	.831	.512	1,674.7	184.4	7.70	206.8	35.2	2.71

Table 6. Properties of WF Sections (Continued)



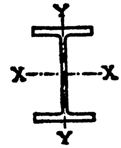
Nominal Size	Weight per Foot	Area of Section	Depth of Section	Flange		Web Thickness	Axis X–X			Axis Y–Y		
				Width	Thickness		I	S	r	I	S	r
In	Lb	In²	In	In	In	In	In⁴	In³	In	In⁴	In³	In
18 x 8¾	70	20.56	18.00	8.750	.751	.438	1,153.9	128.2	7.49	78.5	17.9	1.95
	64	18.80	17.87	8.715	.686	.403	1,045.8	117.0	7.46	70.3	16.1	1.93
18 x 7½	55	16.19	18.12	7.532	.630	.390	889.9	98.2	7.41	42.0	11.1	1.61
	50	14.71	18.00	7.500	.570	.358	800.6	89.0	7.38	37.2	9.9	1.59
16 x 11½	96	28.22	16.32	11.533	.875	.535	1,355.1	166.1	6.93	207.2	35.9	2.71
	88	25.87	16.16	11.502	.795	.504	1,222.6	151.3	6.87	185.2	32.2	2.67
16 x 8½	64	18.80	16.00	8.500	.715	.443	833.8	104.2	6.66	68.4	16.1	1.91
	58	17.04	15.86	8.464	.645	.407	746.4	94.1	6.62	60.5	14.3	1.88
16 x 7	45	13.24	16.12	7.039	.563	.346	583.3	72.4	6.64	30.5	8.7	1.52
	40	11.77	16.00	7.000	.503	.307	515.5	64.4	6.62	26.5	7.6	1.50
	36	10.59	15.85	6.992	.428	.299	446.3	56.3	6.49	22.1	6.3	1.45
14 x 16	158	46.47	15.00	15.550	1.188	.730	1,900.6	253.4	6.40	745.0	95.8	4.00
	142	41.85	14.75	15.500	1.063	.680	1,672.2	226.7	6.32	660.1	85.2	3.97
14 x 14½	136	39.98	14.75	14.740	1.063	.660	1,593.0	216.0	6.31	567.7	77.0	3.77
	119	34.99	14.50	14.650	.938	.570	1,373.1	189.4	6.26	491.8	67.1	3.75
	103	30.26	14.25	14.575	.813	.495	1,165.8	163.6	6.21	419.7	57.6	3.72
	95	27.94	14.12	14.545	.748	.465	1,063.5	150.6	6.17	383.7	52.8	3.71
	87	25.56	14.00	14.500	.688	.420	966.9	138.1	6.15	349.7	48.2	3.70
14 x 12	84	24.71	14.18	12.023	.778	.451	928.4	130.9	6.13	225.5	37.5	3.02
	78	22.94	14.06	12.000	.718	.428	851.2	121.1	6.09	206.9	34.5	3.00
14 x 10	74	21.76	14.19	10.072	.783	.450	796.8	112.3	6.05	133.5	26.5	2.48
	68	20.00	14.06	10.040	.718	.418	724.1	103.0	6.02	121.2	24.1	2.46
	61	17.94	13.91	10.000	.643	.378	641.5	92.2	5.98	107.3	21.5	2.45
14 x 8	53	15.59	13.94	8.062	.658	.370	542.1	77.8	5.90	57.5	14.3	1.92
	48	14.11	13.81	8.031	.593	.339	484.9	70.2	5.86	51.3	12.8	1.91
	43	12.65	13.68	8.000	.528	.308	429.0	62.7	5.82	45.1	11.3	1.89
14 x 6¾	38	11.17	14.12	6.776	.513	.313	385.3	54.6	5.87	24.6	7.3	1.49
	34	10.00	14.00	6.750	.453	.287	339.2	48.5	5.83	21.3	6.3	1.46
	30	8.81	13.86	6.733	.383	.270	289.6	41.8	5.73	17.5	5.2	1.41
12 x 12	190	55.86	14.38	12.670	1.736	1.060	1,892.5	263.2	5.82	589.7	93.1	3.25
	161	47.38	13.88	12.515	1.486	.905	1,541.8	222.2	5.70	486.2	77.7	3.20
	133	39.11	13.38	12.365	1.236	.755	1,221.2	182.5	5.59	389.9	63.1	3.16
	106	31.19	12.88	12.230	.986	.620	930.7	144.5	5.46	300.9	49.2	3.11
	92	27.06	12.62	12.155	.856	.545	788.9	125.0	5.40	256.4	42.2	3.08
	79	23.22	12.38	12.080	.736	.470	663.0	107.1	5.34	216.4	35.8	3.05
	72	21.16	12.25	12.040	.671	.430	597.4	97.5	5.31	195.3	32.4	3.04
	65	19.11	12.12	12.000	.606	.390	533.4	88.0	5.28	174.6	29.1	3.02

Table 6. Properties of WF Sections (Continued)



Nominal Size	Weight per Foot	Area of Section	Depth of Section	Flange		Web Thickness	Axis X-X			Axis Y-Y		
				Width	Thickness		I	S	r	I	S	r
In	Lb	In^2	In	In	In	In	In^4	In^3	In	In^4	In^3	In
12 x 10	58	17.06	12.19	10.014	.641	.359	476.1	78.1	5.28	107.4	21.4	2.51
	53	15.59	12.06	10.000	.576	.345	426.2	70.7	5.23	96.1	19.2	2.48
12 x 8	45	13.24	12.06	8.042	.576	.336	350.8	58.2	5.15	50.0	12.4	1.94
	40	11.77	11.94	8.000	.516	.294	310.1	51.9	5.13	44.1	11.0	1.94
12 x 6½	36	10.59	12.24	6.565	.540	.305	280.8	45.9	5.15	23.7	7.2	1.50
	32	9.41	12.12	6.533	.480	.273	246.8	40.7	5.12	20.6	6.3	1.48
	28	8.23	12.00	6.500	.420	.240	213.5	35.6	5.09	17.5	5.4	1.46
	25	7.39	11.87	6.500	.355	.240	183.4	30.9	4.98	14.5	4.5	1.40
10 x 10	112	32.92	11.38	10.415	1.248	.755	718.7	126.3	4.67	235.4	45.2	2.67
	100	29.43	11.12	10.345	1.118	.685	625.0	112.4	4.61	206.6	39.9	2.65
	89	26.19	10.88	10.275	.998	.615	542.4	99.7	4.55	180.6	35.2	2.63
	77	22.67	10.62	10.195	.868	.535	457.2	86.1	4.49	153.4	30.1	2.60
	66	19.41	10.38	10.117	.748	.457	382.5	73.7	4.44	129.2	25.5	2.58
	54	15.88	10.12	10.028	.618	.368	305.7	60.4	4.39	103.9	20.7	2.56
	49	14.40	10.00	10.000	.558	.340	272.9	54.6	4.35	93.0	18.6	2.54
10 x 8	41	12.06	10.00	8.000	.558	.328	222.4	44.5	4.29	47.7	11.9	1.99
	33	9.71	9.75	7.964	.433	.292	170.9	35.0	4.20	36.5	9.2	1.94
10 x 5¾	26	7.65	10.12	5.769	.450	.259	139.7	27.6	4.27	13.4	4.6	1.32
	21	6.19	9.90	5.750	.340	.240	106.3	21.5	4.14	9.7	3.4	1.25
8 x 8	67	19.70	9.00	8.287	.933	.575	271.8	60.4	3.71	88.6	21.4	2.12
	58	17.06	8.75	8.222	.808	.510	227.3	52.0	3.65	74.9	18.2	2.10
	48	14.11	8.50	8.117	.683	.405	183.7	43.2	3.61	60.9	15.0	2.08
	40	11.76	8.25	8.077	.558	.365	146.3	35.5	3.53	49.0	12.1	2.04
	35	10.30	8.12	8.027	.493	.315	126.5	31.1	3.50	42.5	10.6	2.03
	31	9.12	8.00	8.000	.433	.288	109.7	27.4	3.47	37.0	9.2	2.01
8 x 6½	27	7.93	8.03	6.528	.448	.273	94.1	23.4	3.44	20.8	6.4	1.62
	24	7.06	7.93	6.500	.398	.245	82.5	20.8	3.42	18.2	5.6	1.61
8 x 5¼	21	6.18	8.19	5.272	.403	.252	73.8	18.0	3.45	9.13	3.5	1.22
	17	5.00	8.00	5.250	.308	.230	56.4	14.1	3.36	6.72	2.6	1.16

Table 7. Properties of Miscellaneous Columns and Beams

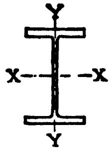


Nominal Size	Weight per Foot	Area of Section	Depth of Section	Flange		Web Thickness	Axis X–X			Axis Y–Y		
				Width	Thickness		I	S	r	I	S	r
In	Lb	In²	In	In	In	In	In⁴	In³	In	In⁴	In³	In
					WF Sections and Light Columns							
6WF *6 x 6	27.5	8.11	6.46	6.112	.500	.352	59.7	18.5	2.71	19.1	6.2	1.53
	25	7.37	6.37	6.080	.456	.320	53.5	16.8	2.69	17.1	5.6	1.52
	22.5	6.63	6.28	6.050	.411	.290	47.4	15.1	2.67	15.2	5.0	1.51
	20	5.90	6.20	6.018	.367	.258	41.7	13.4	2.66	13.3	4.4	1.50
	18	5.31	6.11	6.010	.322	.250	36.4	11.9	2.62	11.7	3.9	1.48
	15.5	4.62	6.00	6.000	.269	.240	30.3	10.1	2.56	9.69	3.2	1.45
5WF *5 x 5	18.5	5.45	5.12	5.025	.420	.265	25.4	9.94	2.16	8.89	3.54	1.28
	16	4.70	5.00	5.000	.360	.240	21.3	8.53	2.13	7.51	3.00	1.26
	13.5	3.98	4.86	4.990	.292	.230	17.1	7.02	2.07	6.05	2.43	1.23
†4 x 4	13	3.82	4.16	4.060	.345	.280	11.3	5.45	1.72	3.76	1.85	.99
	10	2.93	4.00	4.000	.265	.220	8.31	4.16	1.68	2.74	1.37	.97
	7½	2.22	3.87	3.950	.200	.170	6.06	3.13	1.65	1.96	.99	.94
					Light Beams							
*12 x 4	22	6.47	12.31	4.030	.424	.260	155.7	25.3	4.91	4.55	2.26	.84
	19	5.62	12.16	4.010	.349	.240	130.1	21.4	4.81	3.67	1.83	.81
	16½	4.86	12.00	4.000	.269	.230	105.3	17.5	4.65	2.79	1.39	.76
*10 x 4	19	5.61	10.25	4.020	.394	.250	96.2	18.8	4.14	4.19	2.08	.86
	17	4.98	10.12	4.010	.329	.240	81.8	16.2	4.05	3.45	1.72	.83
	15	4.40	10.00	4.000	.269	.230	68.8	13.8	3.95	2.79	1.39	.80
* 8 x 4	15	4.43	8.12	4.015	.314	.245	48.0	11.8	3.29	3.30	1.65	.86
	13	3.83	8.00	4.000	.254	.230	39.5	9.88	3.21	2.62	1.31	.83
* 6 x 4	16	4.72	6.25	4.030	.404	.260	31.7	10.1	2.59	4.32	2.14	.96
	12	3.53	6.00	4.000	.279	.230	21.7	7.24	2.48	2.89	1.44	.90
					Joists							
*12 x 4	14	4.14	11.91	3.970	.224	.200	88.2	14.8	4.61	2.25	1.13	.74
*10 x 4	11½	3.39	9.87	3.950	.204	.180	51.9	10.5	3.92	2.01	1.02	.77
* 8 x 4	10	2.95	7.90	3.940	.204	.170	30.8	7.79	3.23	1.99	1.01	.82
* 6 x 4	8½	2.50	5.83	3.940	.194	.170	14.8	5.07	2.43	1.89	.96	.87

*** Rolled by Bethlehem Steel Co. and Carnegie-Illinois Steel Corp.**
† Rolled by Bethlehem Steel Co.

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Table 8. Properties of Miscellaneous Columns and Beams

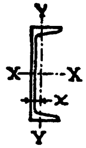


Nominal Size	Weight per Foot	Area of Section	Depth of Section	Width of Flange	Web Thickness	Axis $X-X$			Axis $Y-Y$		
						I	S	r	I	S	r
In	Lb	In²	In	In	In	In⁴	In³	In	In⁴	In³	In
LIGHT COLUMNS											
* 8 x 8	37.7	11.09	8.00	8.125	.500	120.8	30.2	3.31	36.9	9.1	1.83
	34.3	10.09	8.00	8.000	.375	115.5	28.9	3.40	35.1	8.8	1 87
	32.6	9.59	8.00	7.938	.313	112.8	28.2	3.45	34.2	8.6	1.90
* 6 x 6	27.5	8.09	6.00	6.063	.438	49.3	16.4	2.47	16.0	5.3	1.41
	25.0	7.35	6.00	5 938	.313	47.0	15.7	2.53	14.9	5.0	1.43
	22.5	6.62	6.00	6 063	.375	41.0	13.7	2.49	12.2	4.0	1.36
	20.0	5.88	6.00	5.938	.250	38.8	12.9	2.57	11.4	3.8	1.39
† 5 x 5	18.9	5.47	5.00	5.000	.313	23.8	9.5	2.08	7.8	3.1	1.20
‡ 4 x 4	13.8	3.99	4.00	4.000	.313	10.7	5.3	1.64	3.6	1.8	.95
STANDARD MILL BEAMS											
§15 x 5½	36.0	10.59	15.00	5.56	.34	381.5	50.9	6.00	12.0	4.33	1.07
	33.0	9.71	15.00	5.50	.28	365.0	48.7	6.13	11.6	4.22	1.09
§14 x 6¾	34.0	10.00	13.88	6.835	.350	313.8	45.2	5.60	17.5	5.13	1.32
	30.0	8.82	13.88	6.750	.265	294.9	42.5	5.76	16.8	4.99	1.38
§12 x 6½	28.0	8.24	11.88	6.569	.314	193.6	32.6	4.85	13.9	4.28	1.30
	25.0	7.35	11.88	6.495	.240	182.8	30.8	4.98	13.4	4.12	1.35
§10 x 5¾	23.0	6.77	9.90	5.79	.29	112.1	22.6	4.09	9.57	3.30	1.19
	21.0	6.18	9.90	5.74	.24	107.5	21.7	4.17	9.30	3.24	1.22
§ 8 x 5¼	21.0	6.18	8 00	5.40	.38	62.3	15.6	3.18	6.80	2.52	1.05
	19.0	5.59	8.00	5.32	.31	59.2	14.8	3.26	6.45	2.42	1.08
	17.0	5.00	8.00	5.25	.24	56.0	14.0	3.35	6.16	2.35	1.11
† 7 x 3½	12.0	3.52	7.00	3.50	.188	29.8	8.5	2.91	2.1	1.18	.77
† 6 x 3	10.0	2.91	6.00	3.00	.188	17.8	5.9	2.47	1.3	.85	.66
JUNIOR BEAMS											
¶12 x 3	11.8	3.45	12.00	3.063	.175	72.2	12.0	4.57	.98	.64	.53
¶11 x 2⅞	10.3	3.01	11.00	2.844	.165	53.1	9.6	4.20	.75	.52	.50
¶10 x 2¾	9.0	2.64	10.00	2.688	.155	39.0	7.8	3.85	.61	.45	.48
¶ 9 x 2⅜	7.5	2.20	9.00	2.375	.145	26.2	5.8	3.45	.39	.33	.42
¶ 8 x 2¼	6.5	1.92	8.00	2.281	.135	18.7	4.7	3.12	.34	.30	.42
¶ 7 x 2⅛	5.5	1.61	7.00	2.078	.126	12.1	3.5	2.74	.25	.24	.39
¶ 6 x 1⅞	4.4	1.30	6.00	1.844	.114	7.3	2.4	2.37	.17	.18	.36

* Rolled by Carnegie-Illinois Steel Corp., Inland Steel Co. and The Phoenix Iron Co.
† Rolled by Carnegie-Illinois Steel Corp. and Bethlehem Steel Co.
‡ Rolled by Carnegie-Illinois Steel Corp.
§ Rolled by The Phoenix Iron Co.
¶ Rolled by Jones & Laughlin Steel Corp.

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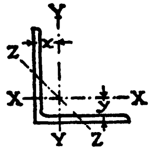
Table 9. Properties of American Standard Channels



Nominal Size	Weight per Foot	Area of Section	Depth of Section	Width of Flange	Web Thickness	Axis X–X			Axis Y–Y			
						I	*S*	*r*	*I*	*S*	*r*	*x*
In	Lb	In²	In	In	In	In⁴	In³	In	In⁴	In³	In	In
15 x 3⅜	55.0	16.11	15.00	3.814	.814	429.0	57.2	5.16	12.1	4.1	.87	.82
	50.0	14.64	15.00	3.716	.716	401.4	53.6	5.24	11.2	3.8	.87	.80
	45.0	13.17	15.00	3.618	.618	373.9	49.8	5.33	10.3	3.6	.88	.79
	40.0	11.70	15.00	3.520	.520	346.3	46.2	5.44	9.3	3.4	.89	.78
	35.0	10.23	15.00	3.422	.422	318.7	42.5	5.58	8.4	3.2	.91	.79
	33.9	9.90	15.00	3.400	.400	312.6	41.7	5.62	8.2	3.2	.91	.79
12 x 3	40.0	11.73	12.00	3.415	.755	196.5	32.8	4.09	6.6	2.5	.75	.72
	35.0	10.26	12.00	3.292	.632	178.8	29.8	4.18	5.9	2.3	.76	.69
	30.0	8.79	12.00	3.170	.510	161.2	26.9	4.28	5.2	2.1	.77	.63
	25.0	7.32	12.00	3.047	.387	143.5	23.9	4.43	4.5	1.9	.79	.68
	20.7	6.03	12.00	2.940	.280	128.1	21.4	4.61	3.9	1.7	.81	.70
10 x 2⅝	35.0	10.27	10.00	3.180	.820	115.2	23.0	3.34	4.6	1.9	.67	.69
	30.0	8.80	10.00	3.033	.673	103.0	20.6	3.42	4.0	1.7	.67	.65
	25.0	7.33	10.00	2.886	.526	90.7	18.1	3.52	3.4	1.5	.68	.62
	20.0	5.86	10.00	2.739	.379	78.5	15.7	3.66	2.8	1.3	.70	.61
	15.3	4.47	10.00	2.600	.240	66.9	13.4	3.87	2.3	1.2	.72	.64
9 x 2½	25.0	7.33	9.00	2.812	.612	70.5	15.7	3.10	3.0	1.4	.64	.61
	20.0	5.86	9.00	2.648	.448	60.6	13.5	3.22	2.4	1.2	.65	.59
	15.0	4.39	9.00	2.485	.285	50.7	11.3	3.40	1.9	1.0	.67	.59
	13.4	3.89	9.00	2.430	.230	47.3	10.5	3.49	1.8	.97	.67	.61
8 x 2¼	21.25	6.23	8.00	2.619	.579	47 6	11.9	2.77	2.2	1.1	.60	.59
	18.75	5.49	8.00	2.527	.487	43.7	10.9	2.82	2.0	1.0	.60	.57
	16.25	4.76	8.00	2.435	.395	39.8	9.9	2.89	1.8	.94	.61	.56
	13.75	4.02	8.00	2 343	.303	35.8	9.0	2.99	1.5	.86	.62	.56
	11.5	3.36	8.00	2.260	.220	32.3	8.1	3.10	1.3	.79	.63	.58
7 x 2⅛	19.75	5.79	7.00	2.509	.629	33.1	9.4	2.39	1.8	.96	.56	.58
	17.25	5.05	7.00	2.404	.524	30.1	8.6	2.44	1.6	.86	.56	.55
	14.75	4.32	7.00	2.299	.419	27.1	7.7	2.51	1.4	.79	.57	.53
	12.25	3.58	7.00	2.194	.314	24.1	6.9	2.59	1.2	.71	.58	.53
	9.8	2.85	7.00	2.090	.210	21.1	6.0	2.72	.98	.63	.59	.55
6 x 2	15.5	4.54	6.00	2.279	.559	19.5	6.5	2.07	1.3	.73	.53	.55
	13.0	3.81	6.00	2.157	.437	17.3	5.8	2.13	1.1	.65	.53	.52
	10.5	3.07	6.00	2.034	.314	15.1	5.0	2.22	.87	.57	.53	.50
	8.2	2.39	6.00	1.920	.200	13.0	4.3	2.34	.70	.50	.54	.52
5 x 1¾	11.5	3.36	5.00	2.032	.472	10.4	4.1	1.76	.82	.54	.49	.51
	9.0	2.63	5.00	1.885	.325	8.8	3.5	1.83	.64	.45	.49	.48
	6.7	1.95	5.00	1.750	.190	7.4	3.0	1.95	.48	.38	.50	.49
4 x 1⅝	7.25	2.12	4.00	1.720	.320	4.5	2.3	1.47	.44	.35	.46	.46
	6.25	1.82	4.00	1.647	.247	4.1	2.1	1.50	.38	.32	.45	.46
	5.4	1.56	4.00	1.580	.180	3.8	1.9	1.56	.32	.29	.45	.46
3 x 1½	6.0	1.75	3.00	1.596	.356	2.1	1.4	1.08	.31	.27	.42	.46
	5.0	1.46	3.00	1.498	.258	1.8	1.2	1.12	.25	.24	.41	.44
	4.1	1.19	3.00	1.410	.170	1.6	1.1	1.17	.20	.21	.41	.44

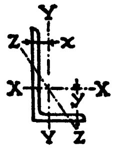
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Table 10. Properties of Angles with Equal Legs



Size	Thickness	Weight per Foot	Area of Section	Axis X–X and Axis Y–Y				Axis Z–Z
				I	*S*	*r*	*x* or *y*	*r*
In	In	Lb	In²	In⁴	In³	In	In	In
8 x 8	1⅛	56.9	16.73	98.0	17.5	2.42	2.41	1.56
	1	51.0	15.00	89.0	15.8	2.44	2.37	1.56
	⅞	45.0	13.23	79.6	14.0	2.45	2.32	1.57
	¾	38.9	11.44	69.7	12.2	2.47	2.28	1.57
	⅝	32.7	9.61	59.4	10.3	2.49	2.23	1.58
	½	26.4	7.75	48.6	8.4	2.50	2.19	1.59
6 x 6	1	37.4	11.00	35.5	8.6	1.80	1.86	1.17
	⅞	33.1	9.73	31.9	7.6	1.81	1.82	1.17
	¾	28.7	8.44	28.2	6.7	1.83	1.78	1.17
	⅝	24.2	7.11	24.2	5.7	1.84	1.73	1.18
	½	19.6	5.75	19.9	4.6	1.86	1.68	1.18
	7/16	17.2	5.06	17.7	4.1	1.87	1.66	1.19
	⅜	14.9	4.36	15.4	3.5	1.88	1.64	1.19
5 x 5	1	30.6	9.00	19.6	5.8	1.48	1.61	.97
	⅞	27.2	7.98	17.8	5.2	1.49	1.57	.97
	¾	23.6	6.94	15.7	4.5	1.51	1.52	.97
	⅝	20.0	5.86	13.6	3.9	1.52	1.48	.98
	½	16.2	4.75	11.3	3.2	1.54	1.43	.98
	7/16	14.3	4.18	10.0	2.8	1.55	1.41	.98
	⅜	12.3	3.61	8.7	2.4	1.56	1.39	.99
4 x 4	¾	18.5	5.44	7.7	2.8	1.19	1.27	.78
	⅝	15.7	4.61	6.7	2.4	1.20	1.23	.78
	½	12.8	3.75	5.6	2.0	1.22	1.18	.78
	7/16	11.3	3.31	5.0	1.8	1.23	1.16	.78
	⅜	9.8	2.86	4.4	1.5	1.23	1.14	.79
	5/16	8.2	2.40	3.7	1.3	1.24	1.12	.79
	¼	6.6	1.94	3.0	1.1	1.25	1.09	.80
3½ x 3½	¾	16.0	4.69	5.0	2.1	1.03	1.15	.68
	⅝	13.6	3.98	4.3	1.8	1.04	1.10	.68
	½	11.1	3.25	3.6	1.5	1.06	1.06	.68
	7/16	9.8	2.87	3.3	1.3	1.07	1.04	.68
	⅜	8.5	2.48	2.9	1.2	1.07	1.01	.69
	5/16	7.2	2.09	2.5	.98	1.08	.99	.69
	¼	5.8	1.69	2.0	.79	1.09	.97	.69
3 x 3	⅝	11.5	3.36	2.6	1.3	.88	.98	.58
	½	9.4	2.75	2.2	1.1	.90	.93	.58
	⅜	7.2	2.11	1.8	.83	.91	.89	.58
	5/16	6.1	1.78	1.5	.71	.92	.87	.59
	¼	4.9	1.44	1.2	.58	.93	.84	.59
2½ x 2½	½	7.7	2.25	1.2	.72	.74	.81	.49
	⅜	5.9	1.73	.98	.57	.75	.76	.49
	5/16	5.0	1.47	.85	.48	.76	.74	.49
	¼	4.1	1.19	.70	.39	.77	.72	.49
	3/16	3.07	.90	.55	.30	.78	.69	.49
2 x 2	⅜	4.7	1.36	.48	.35	.59	.64	.39
	5/16	3.92	1.15	.42	.30	.60	.61	.39
	¼	3.19	.94	.35	.25	.61	.59	.39
	3/16	2.44	.71	.27	.19	.62	.57	.39

Table 11. Properties of Angles with Unequal Legs



Size	Thickness	Weight per Foot	Area of Section	Axis X–X				Axis Y–Y				Axis Z–Z
				I	*S*	*r*	*y*	*I*	*S*	*r*	*x*	*r*
In	In	Lb	In²	In⁴	In³	In	In	In⁴	In³	In	In	In
8 x 6	1⅛	49.3	14.48	88.9	16.8	2.48	2.70	42.5	9.9	1.71	1.70	1.28
	¾	33.8	9.94	63.4	11.7	2.53	2.56	30.7	6.9	1.76	1.56	1.29
	½	23.0	6.75	44.3	8.0	2.56	2.47	21.7	4.8	1.79	1.47	1.30
8 x 4	1	37.4	11.00	69.6	14.1	2.52	3.05	11.6	3.9	1.03	1.05	.85
	¾	28.7	8.44	54.9	10.9	2.55	2.95	9.4	3.1	1.05	.95	.85
	⅝	24.2	7.11	46.9	9.2	2.57	2.91	8.1	2.6	1.07	.91	.86
	½	19.6	5.75	38.5	7.5	2.59	2.86	6.7	2.2	1.08	.86	.86
7 x 4	⅞	30.2	8.86	42.9	9.7	2.20	2.55	10.2	3.5	1.07	1.05	.86
	¾	26.2	7.69	37.8	8.4	2.22	2.51	9.1	3.0	1.09	1.01	.86
	⅝	22.1	6.48	32.4	7.1	2.24	2.46	7.8	2.6	1.10	.96	.86
	½	17.9	5.25	26.7	5.8	2.25	2.42	6.5	2.1	1.11	.92	.87
	⅜	13.6	3.98	20.6	4.4	2.27	2.37	5.1	1.6	1.13	.87	.88
6 x 4	⅞	27.2	7.98	27.7	7.2	1.86	2.12	9.8	3.4	1.11	1.12	.86
	¾	23.6	6.94	24.5	6.3	1.88	2.08	8.7	3.0	1.12	1.08	.86
	⅝	20.0	5.86	21.1	5.3	1.90	2.03	7.5	2.5	1.13	1.03	.86
	½	16.2	4.75	17.4	4.3	1.91	1.99	6.3	2.1	1.15	.99	.87
	7/16	14.3	4.18	15.5	3.8	1.92	1.96	5.6	1.9	1.16	.96	.87
	⅜	12.3	3.61	13.5	3.3	1.93	1.94	4.9	1.6	1.17	.94	.88
6 x 3½	¾	22.4	6.56	23.3	6.1	1.89	2.18	5.8	2.3	.94	.93	.75
	⅝	18.9	5.55	20.1	5.2	1.90	2.13	5.1	1.9	.96	.88	.75
	½	15.3	4.50	16.6	4.2	1.92	2.08	4.3	1.6	.97	.83	.76
	⅜	11.7	3.42	12.9	3.3	1.94	2.04	3.3	1.2	.99	.79	.77
5 x 3½	¾	19.8	5.81	13.9	4.3	1.55	1.75	5.6	2.2	.98	1.00	.75
	⅝	16.8	4.92	12.0	3.7	1.56	1.70	4.8	1.9	.99	.95	.75
	½	13.6	4.00	10.0	3.0	1.58	1.66	4.1	1.6	1.01	.91	.75
	⅜	10.4	3.05	7.8	2.3	1.60	1.61	3.2	1.2	1.02	.86	.76
5 x 3	¾	18.5	5.44	13.2	4.2	1.55	1.84	3.5	1.6	.80	.84	.64
	⅝	15.7	4.61	11.4	3.5	1.57	1.80	3.1	1.4	.81	.80	.64
	½	12.8	3.75	9.5	2.9	1.59	1.75	2.6	1.1	.83	.75	.65
	⅜	9.8	2.86	7.4	2.2	1.61	1.70	2.0	.89	.84	.70	.65
4 x 3½	⅝	14.7	4.30	6.4	2.4	1.22	1.29	4.5	1.8	1.03	1.04	.72
	½	11.9	3.50	5.3	1.9	1.23	1.25	3.8	1.5	1.04	1.00	.72
	⅜	9.1	2.67	4.2	1.5	1.25	1.21	3.0	1.2	1.06	.96	.73
	5/16	7.7	2.25	3.6	1.3	1.26	1.18	2.6	1.0	1.07	.93	.73
4 x 3	⅝	13.6	3.98	6.0	2.3	1.23	1.37	2.9	1.4	.85	.87	.64
	½	11.1	3.25	5.1	1.9	1.25	1.33	2.4	1.1	.86	.83	.64
	⅜	8.5	2.48	4.0	1.5	1.26	1.28	1.9	.87	.88	.78	.64
	5/16	7.2	2.09	3.4	1.2	1.27	1.26	1.7	.73	.89	.76	.65
	¼	5.8	1.69	2.8	1.0	1.28	1.24	1.4	.60	.90	.74	.65
3½ x 3	⅝	12.5	3.67	4.1	1.8	1.06	1.17	2.8	1.3	.87	.92	.62
	½	10.2	3.00	3.5	1.5	1.07	1.13	2.3	1.1	.88	.88	.62
	⅜	7.9	2.30	2.7	1.1	1.09	1.08	1.9	.85	.90	.83	.62
	5/16	6.6	1.93	2.3	.95	1.10	1.06	1.6	.72	.90	.81	.63
	¼	5.4	1.56	1.9	.78	1.11	1.04	1.3	.59	.91	.79	.63
3 x 2½	½	8.5	2.50	2.1	1.0	.91	1.00	1.3	.74	.72	.75	.52
	⅜	6.6	1.92	1.7	.81	.93	.96	1.0	.58	.74	.71	.52
	5/16	5.6	1.62	1.4	.69	.94	.93	.90	.49	.74	.68	.53
	¼	4.5	1.31	1.2	.56	.95	.91	.74	.40	.75	.66	.53

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Table 12. Properties of American Standard Yard Lumber and Timber Sizes *

Size	American Standard Dressed Size	Area of Section	Weight per Linear Foot †	Moment of Inertia	Section Modulus
(Nominal in inches)	In	$A = bd$ In²	Lb	$I = \frac{bd^3}{12}$	$S = \frac{bd^2}{6}$
2 x 4	1⅝ x 3⅝	5.89	1.6	6.45	3.56
2 x 6	1⅝ x 5⅝	9.14	2.5	24.10	8.57
2 x 8	1⅝ x 7½	12.19	3.4	57.13	15.32
2 x 10	1⅝ x 9½	15.44	4.3	116.09	24.44
2 x 12	1⅝ x 11½	18.69	5.2	205.94	35.82
2 x 14	1⅝ x 13½	23.62	6.5	333.15	49.36
3 x 4	2⅝ x 3⅝	9.51	2.6	10.42	5.75
3 x 6	2⅝ x 5⅝	14.76	4.2	38.93	13.84
3 x 8	2⅝ x 7½	19.68	5.7	92.28	24.60
3 x 10	2⅝ x 9½	24.93	7.2	187.55	39.48
3 x 12	2⅝ x 11½	30.18	8.8	332.69	57.86
3 x 14	2⅝ x 13½	35.43	10.3	538.21	79.73
4 x 4	3⅝ x 3⅝	13.14	3.6	14.38	7.94
4 x 6	3⅝ x 5⅝	20.39	5.7	53.76	19.11
4 x 8	3⅝ x 7½	27.18	7.5	127.44	33.98
4 x 10	3⅝ x 9½	34.43	9.6	258.99	54.52
4 x 12	3⅝ x 11½	41.68	11.6	459.42	79.90
4 x 14	3⅝ x 13½	48.93	13.6	743.23	110.11
4 x 16	3⅝ x 15½	56.18	15.6	1,124.90	145.15
6 x 6	5½ x 5½	30.25	8.4	76.25	27.73
6 x 8	5½ x 7½	41.25	11.4	193.35	51.56
6 x 10	5½ x 9½	52.25	14.5	392.96	82.73
6 x 12	5½ x 11½	63.25	17.5	697.06	121.23
6 x 14	5½ x 13½	74.25	20.6	1,127.66	167.06
6 x 16	5½ x 15½	85.25	23.6	1,706.76	220.22
8 x 8	7½ x 7½	56.25	15.6	263.67	70.31
8 x 10	7½ x 9½	71.25	19.8	535.85	112.81

* Compiled from data published by the United States Department of Agriculture, Forest Products Laboratory.

† Based on assumed average weight of 40#/ft³.

Table 12 (Continued)

Size	American Standard Dressed Size	Area of Section	Weight per Linear Foot †	Moment of Inertia	Section Modulus
(Nominal in inches)	In	$A = bd$ In²	Lb	$I = \frac{bd^3}{12}$	$S = \frac{bd^2}{6}$
8 x 12	7½ x 11½	86.25	23.9	950.55	165.31
8 x 14	7½ x 13½	101.25	28.0	1,537.73	227.81
8 x 16	7½ x 15½	116.25	32.0	2,327.42	300.31
10 x 10	9½ x 9½	90.25	25.0	678.75	142.89
10 x 12	9½ x 11½	109.25	30.3	1,204.01	209.39
10 x 14	9½ x 13½	128.25	35.6	1,947.78	288.56
10 x 16	9½ x 15½	147.25	40.9	2,948.04	380.39
12 x 12	11½ x 11½	132.25	36.7	1,457.50	253.47
12 x 14	11½ x 13½	155.25	43.1	2,357.85	349.31
12 x 16	11½ x 15½	178.25	49.5	3,568.70	460.48
12 x 18	11½ x 17½	201.25	55.9	5,136.49	586.98

† Based on assumed average weight of 40#/ft³.

SECTION II

TIMBER CONSTRUCTION

46. **Dead and Live Loads.** In designing a beam for a given position in a structure the first step is to estimate the greatest load which it may be required to support. The total load is composed of the dead load, which comprises the materials of construction (such as floor materials, partitions, columns, and walls), and the live load, or that load due to human occupants, furniture, equipment, stored materials, snow, and movable partitions. The type of construction being known, the dead load may be readily computed by referring to Tables 13 and 14. The live load to be used in computations depends upon the use or occupancy of the building. Building codes of cities differ greatly in their requirements for minimum live loads. Table 15 gives such loads as required by several representative cities as well as recommendations of the U. S. Department of Commerce.

In certain classes of buildings, office buildings for example, there is always a possibility of the relocation of partitions. A general practice among designers is to add from 15 to 20#/ft^2 for *movable partitions* in tabulating the floor loads.

47. **Horizontal Shear.** In every beam there is a tendency to fail by vertical shear as illustrated in Fig. 16 (*b*). The magnitude of the vertical shear V for simple beams is equal to the greatest reaction, and for beams symmetrically loaded is equal, of course, to ½ the total load. It is assumed that the vertical shearing unit stresses are equally distributed over the cross section, and that for rectangular beams the vertical shearing unit stress equals $\frac{V}{bd}$.

However, beams are more likely to fail because of the tendency of the fibers to slide in a horizontal direction as illustrated in Fig. 16 (*d*). This is known as horizontal shear. The stresses are not equally distributed over the cross section but are greatest at the neutral surface, the maximum horizontal shearing unit stress, for

Table 13. Weights of Building Materials

Materials	Pounds per Cubic Foot	Pounds per Square Foot
Ashlar masonry, granite, gneiss	165	....
Ashlar masonry, limestone, marble	160	....
Ashlar masonry, sandstone, bluestone	140	....
Cinder-concrete fill	60	....
Cinder-concrete floor-arches	105	....
Cinder fill, rammed in place	50 to 55	....
Plain stone or gravel concrete	144	....
Reinforced stone or gravel concrete	150	....
Common brickwork	120	....
Pressed brickwork	140	......
Brickwork, 4″ with 4″ tile backing	......	60
Brickwork, 4″ with 8″ tile backing	....	75
Cement-mortar finish, 1″ thick	....	12
Clay-tile partitions, 3″	.....	18
Clay-tile partitions, 4″	....	19
Clay-tile partitions, 6″	....	25
Clay-tile partitions, 8″	....	31
Creosoted wood-block flooring, 3″ thick	...	15
Gypsum partitions, 3″ hollow	......	11½
Gypsum partitions, 4″ hollow	....	14½
Wood flooring, ⅞″ thick	......	3
Lime or gypsum plasters, ¾″ thick	......	5
Pine, spruce, or hemlock sheathing	....	2½
Roofing-felt, 3-ply, and gravel	...	5½
Roofing-felt, 4-ply, and gravel	.....	6
Roofing-felt, 5-ply, and gravel	.....	6½
Roofing-felt, 3-ply, and slag	....	4½
Roofing-felt, 4-ply, and slag	.....	5
Roofing-felt, 5-ply, and slag	......	5½
Roofing-tile, laid in place, book-tile 2″	.....	12
Roofing-tile, laid in place, book-tile 3″	....	20
Roofing-tile, laid in place, flat, cement	.....	15 to 20
Roofing-tile, laid in place, shingle-type, clay	......	12 to 14
Roofing-tile, laid in place, Spanish	......	8 to 10
Skylights with ⅜″ wire-glass and frame	....	7½
Slate, laid in place, ¼″	....	9½
Slate, laid in place, ⅜″	.....	14½
Slate, laid in place, ½″	....	19½
Suspended ceilings, metal lath and cement plaster	....	10
Stud partition, 4″, plastered both sides	....	20
Wall-tiles, 8″	....	33
Wall-tiles, 12″	....	45

Table 14. Weights of Floor Materials in Pounds per Square Foot

Board flooring,	per inch of thickness............	3
Granolithic flooring,	" " " "	12
Floor tile,	" " " "	10
Asphalt mastic,	" " " "	12
Wood block,	" " " "	4
Cinder-concrete fill,	" " " "	8
Stone-concrete slab,	" " " "	12
Slag-concrete slab,	" " " "	10
Plaster ceiling on wood lath	....	8
Plaster ceiling on metal lath	....	10
Stamped metal ceiling	....	2

rectangular sections, being $3/2$ times the average vertical shearing unit stress. This may be represented by the formula

$$v = \frac{3}{2} \times \frac{V}{bd}$$

in which v = maximum horizontal shearing unit stress, in pounds per square inch
V = total vertical shear, in pounds
b = breadth of cross section, in inches
d = depth of cross section, in inches.

This formula applies only to rectangular cross sections. Timber is relatively weak in resistance to horizontal shear, and short spans with large loads should be tested for this shearing tendency. Frequently a beam sufficiently large to resist bending stresses must be made larger to resist horizontal shear. Table 3 gives allowable horizontal shearing unit stresses for timber.

EXAMPLE

EXAMPLE. A 4″ x 8″ Southern yellow pine beam, No. 1 Structural grade, has a total uniformly distributed load of 3,600#. Test the beam for horizontal shear using actual sizes.

SOLUTION. $R_1 = R_2 = \frac{3{,}600}{2} = 1{,}800\#$; therefore the maximum vertical shear $V = 1{,}800\#$.

$$v = \frac{3}{2} \times \frac{V}{bd}$$

$$v = \frac{3}{2} \times \frac{1{,}800}{3.625 \times 7.5}$$

$$v = 99\#\text{in}^2$$

Table 15. Minimum Live Loads

Class of Buildings	Minimum Live Load per Square Foot of Floor				
	New York 1938	Phila-delphia 1942	Chicago 1939	Pacific Coast B'ld'g Officials Con-ference 1937	Depart-ment of Com-merce 1924
Dwellings, hotels, tenements, clubhouses, apartments	40	40	40	40	40
Office buildings					
Office space	50	60	50	50	50
Corridors and public spaces	100	100	100	100	100
School classrooms	60	50	50	40	50
Buildings for public assembly					
With fixed seats	75	60	75	50	50
Without fixed seats	100	100	100	100	100
Garages					
All types of vehicles	175	100	100	100	100
Passenger cars only	75	100	100	100	80
Manufacturing buildings					
Heavy		200	100	125	100
Light	120	120	100	75	75
Stairs and fire escapes	100	100	100	100	100
Roofs, flat	40	30	25	20	30
Sidewalks	300	300	150	250	250

The allowable horizontal shear for this timber is 100#/in^2 (Table 3); therefore the beam is acceptable for horizontal shear.

PROBLEMS

47-A. A Douglas fir, Select Structural grade beam has a cross section of 10″ x 10″. A load of 14,000# is concentrated at the center of the span. Test the beam for horizontal shear.

47-B. A simple beam 10″ x 14″ in cross section is loaded symmetrically with three concentrated loads of 4,300# each. If the timber is Eastern hemlock, Select Structural grade, is the beam safe in horizontal shear?

47-C. A 10″ x 12″ S.Y.P., No. 1 Structural grade beam is 8′ in length and has a concentrated load of 8,000# at 3′ from one end. Test the beam for horizontal shear.

47-D. What should be the cross-section dimensions of a simple beam of 1,100# f grade of Southern cypress to resist the horizontal shearing stresses due to a uniformly distributed load of 10,000#?

47-E. An Eastern hemlock, Select Structural grade beam has a 10″ x 12″ cross section. It is symmetrically loaded with four concentrated loads of 5,000# each. Test the beam for horizontal shear.

47-F. A Douglas fir, Dense Select Structural grade beam 18′0″ in length has a 6″ x 10″ cross section. The beam supports a uniformly distributed load of 300 pounds per linear foot. Is it safe with respect to horizontal shear?

47-G. What should be the minimum cross-section area of a simple beam of Douglas fir, Select Structural grade, to resist the horizontal shear resulting from two concentrated loads, symmetrically placed, of 8575# each?

48. **Deflection.** A beam supporting a floor load may be sufficiently large to support the load safely, but the deflection may be so great that a plastered ceiling would develop cracks or the floor might vibrate unduly. That is, a beam may be designed for *strength* or *stiffness*. In practice a beam is designed for bending and then tested for deflection. Most building codes, as well as good practice, require that the maximum deflection does not exceed $\frac{1}{360}$ of the span. For instance, a beam 16′ in length would have an allowable deflection of $\frac{16 \times 12}{360}$ or 0.53″.

A rule of thumb for determining the limiting span with respect to deflection of a timber beam uniformly loaded is: *The limiting span in feet (L) for a simple beam uniformly loaded is equal to 1.1 times the depth of the beam in inches.* It may be expressed thus: $L = 1.1 \times d$. A beam 10″ in depth would have a limiting span of 10×1.1 or 11′.

This rule is, of course, merely approximate, and the actual deflections are found by the use of formulas given in Table 4. The deflection is in inches and is represented by D. In using these formulas note carefully that l, the length of the beam, is in inches.

EXAMPLE

EXAMPLE. Compute the deflection of a 10″ x 14″ L.S.P. beam 15′ in length having a uniformly distributed load of 16,000#.

SOLUTION. The formula for deflection is found in Table 4, Case II. It reads: $D = \frac{5}{384} \times \frac{Wl^3}{EI}$. E for L.S.P. is 1,600,000#/in² (Table 3). Therefore

$$D = \frac{5}{384} \times \frac{16{,}000 \times (15 \times 12)^3}{1{,}600{,}000 \times \frac{10 \times 14^3}{12}}$$

which may be written

$$D = \frac{5 \times 16{,}000 \times 15 \times 15 \times 15 \times 12 \times 12 \times 12 \times 12}{384 \times 1{,}600{,}000 \times 10 \times 14 \times 14 \times 14}$$

$D = 0.33''$, the actual deflection

As the allowable deflection $= \frac{15 \times 12}{360} = 0.5''$, the deflection is not excessive. Note that nominal sizes are used in this example for the purpose of illustration. If actual sizes are used, 9.5″ x 13.5″, the deflection is 0.39″.

If we apply the rule of thumb, the limiting span for a 14″ beam will be $d \times 1.1$, or $14 \times 1.1 = 15.4'$, which of course is greater than the actual span, which is 15′.

PROBLEMS

48-A. A 6″ x 14″ beam for which $E = 1{,}500{,}000\#/\text{in}^2$ is 16′ long and has a uniformly distributed load of 6,000#. Investigate for deflection.

48-B. An 8″ x 12″ S.Y.P. beam 12′ in length has a concentrated load of 5,000# at the center of the span. Compute the actual deflection and compare it with the allowable.

48-C. Two concentrated loads of 3,500# each are located at the third points of a 10″ x 14″ beam 15′ long. If the timber is Douglas fir, investigate the deflection. (NOTE: To find the actual deflection first find an equivalent distributed load producing the same bending moment, or refer to Table 4. See the second example under Art. 49.)

48-D. An 8″ x 12″ longleaf Southern pine, No. 1 Structural grade beam is 15′ 0″ long and has a uniformly distributed load of 8,650#. Is the beam safe for deflection?

48-E. An Eastern spruce beam 8″ x 14″ in cross section has a span of 18′0″ and a uniformly distributed load of 8,000#. Investigate the deflection.

48-F. A 6″ x 14″ Eastern hemlock, Select Structural grade, cantilever beam, 5′0″ in length, has a concentrated load of 3,000# at its free end. Compute the maximum deflection.

49. **Design of Timber Beams.** In general, three steps are necessary for the proper design of timber beams. Actually, however, many designers use only the first, selecting sizes which by experience they know will meet the requirements in Steps 2 and 3. The three steps are as follows.

STEP 1. Compute the section modulus by means of the flexure formula, $\frac{M}{f} = S$. First compute the maximum bending moment and divide by the allowable extreme fiber stress in bending. For allowable stresses see Table 3. This gives the required section modulus. A beam is then selected that has a section modulus equal to or greater than that required. S, the section modulus, is equal to $\frac{I}{c}$, and for rectangular sections $\frac{I}{c} = \frac{bd^2}{6}$ (see Art. 43). Obviously many sizes will meet the requirements, but the most practical sizes have breadths ranging from ½ to ⅓ the depth. They may be selected directly by referring to Table 12. Material which is too thin tends to bend sidewise unless properly braced. Attention has been called to the difference between the nominal size of timber and the actual or dressed size.

STEP 2. Test the size selected under Step 1 for horizontal shear and increase the dimensions if necessary. For allowable horizontal shearing stresses see Table 3.

STEP 3. Test the beam for deflection, the allowable deflection being $\frac{1}{360}$ of the span. Formulas for actual deflection are found in Table 4.

EXAMPLES

Example. Design beam A (Fig. 46), the following specifications being given: live load, 100#/ft^2, 3″ plank under floor, $\frac{7}{8}$″ top floor, timber to be Douglas fir, Select Structural grade.

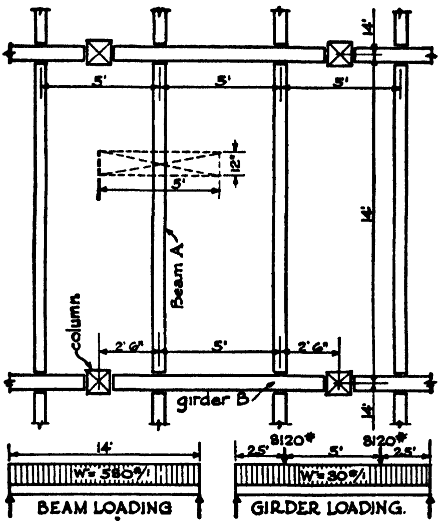


Fig. 46

Solution. Computing the loads,

Computing the loads		Allowable Stresses
Live load	= 100	f = 1,600#/in^2, Table 3
3″ plank	= 9, Table 13	v = 100#/in^2, Table 3
$\frac{7}{8}$″ top floor	= 3, Table 13	E = 1,600,000#/in^2, Table 3
Total	= 112#/ft^2	

Floor load per linear foot of beam = $112 \times 5 = 560$# (floor area marked by diagonals).

Assume beam to be 6″ x 12″. Weight per linear foot $= \dfrac{6 \times 12}{144} \times 40 = 20$#.

Total load on beam per linear foot including its own weight = $560 + 20 = 580$#.

Total load on beam = 580 × 14 = 8,120#. $R_1 = R_2 = \frac{8,120}{2}$ = 4,060#.

STEP 1. Design beam for flexure.

$$M = \frac{wl^2}{8} = \frac{580 \times 14 \times 14}{8} = 14,210'\# = 170,520''\#$$

$$\frac{M}{f} = S = \frac{170,520}{1,600} = 106\text{in}^3$$

$$S = \frac{I}{c} = \frac{bd^2}{6} = 106, \quad bd^2 = 636$$

Assume b to be 5½″; then d = 10″+. A beam 6″ x 12″ is acceptable for flexure.

STEP 2. Test beam for horizontal shear.

Maximum vertical shear equals greatest reaction, or V = 4,060#, $v = \frac{3}{2} \times \frac{V}{bd}$, $v = \frac{3}{2} \times \frac{4,060}{5\frac{1}{2} \times 11\frac{1}{2}}$ = 97#/in². This is acceptable since 100#/in² is allowable (Table 3).

STEP 3. Test for deflection.

Actual deflection, $D = \frac{5}{384} \times \frac{Wl^3}{EI}$, Table 4, Case II.

$$D = \frac{5 \times 8,120 \times 14 \times 12 \times 14 \times 12 \times 14 \times 12 \times 12}{384 \times 1,600,000 \times 5.5 \times 11.5 \times 11.5 \times 11.5} = 0.43'',$$

the actual deflection

Allowable deflection $= \frac{14 \times 12}{360}$ = 0.46″. Since the actual deflection 0.43″ is less than the allowable 0.46″, a 6″ x 12″ beam is acceptable.

EXAMPLE. Design girder B (Fig. 46), using the same specifications given for beam A.

SOLUTION. Assume the girder to be 8″ x 14″; then its weight per linear foot equals $\frac{8 \times 14}{144} \times 40$ = 30#, approximately. The reaction from beam A is 4,060#, and since a beam frames into the girder on each side, the concentrated load will be 2 × 4,060 or

8,120#. The loads on the girder will therefore consist of a uniformly distributed load of 30#/ft and two concentrated loads of 8,120# each, located at 2.5′ from the end supports. See girder-loading diagram (Fig. 46).

STEP 1. Design girder for flexure.

$$R_1 = R_2 = \frac{(2 \times 8{,}120) + (10 \times 30)}{2} = 8{,}270\#$$

The maximum bending moment occurs at the center of the span, at 5′ from the supports.

$$M = (8{,}270 \times 5) - [(30 \times 5 \times 2.5) + (8{,}120 \times 2.5)]$$

$$M = 20{,}675'\# = 248{,}100''\#$$

$$\frac{M}{f} = S = \frac{248{,}100}{1{,}600} = 155\text{in}^3$$

$$S = \frac{I}{c} = \frac{bd^2}{6} = 155,\ bd^2 = 930$$

If $b = 7.5''$, $d = 11.13''$. Therefore try a beam 8″ x 12″.

STEP 2. Test for horizontal shear.

$$V = R_1 = R_2 = 8{,}270\#$$

$$v = \frac{3}{2} \times \frac{V}{bd} = \frac{3}{2} \times \frac{8{,}270}{7.5 \times 11.5} = 143\#/\text{in}^2$$

This is greater than the allowable horizontal shearing stress, which is 100#/in^2.

Try a 10″ x 12″.

$$v = \frac{3}{2} \times \frac{8{,}270}{9.5 \times 11.5} = 113\#/\text{in}^2$$

This stress is excessive; accept a 10″ x 14″.

STEP 3. Test for deflection.

Since this is not a typical loading there is no formula for deflection. It is necessary therefore to find an equivalent uniformly distributed load that would produce the same bending moment.

$$M = 20{,}675'\# \quad \text{(Step 1)}$$

$$M = \frac{Wl}{8}$$

$$20{,}675 = \frac{W \times 10}{8}$$

$W = 16{,}540\#$, the equivalent uniformly distributed load

$$D = \frac{5}{384} \times \frac{Wl^3}{EI}$$

$$D = \frac{5 \times 16{,}540 \times 10 \times 12 \times 10 \times 12 \times 10 \times 12 \times 12}{384 \times 1{,}600{,}000 \times 9.5 \times 13.5 \times 13.5 \times 13.5} = 0.112''$$

The allowable deflection $= \dfrac{10 \times 12}{360} = 0.33''$. Therefore a 10″ x 14″ girder is acceptable.

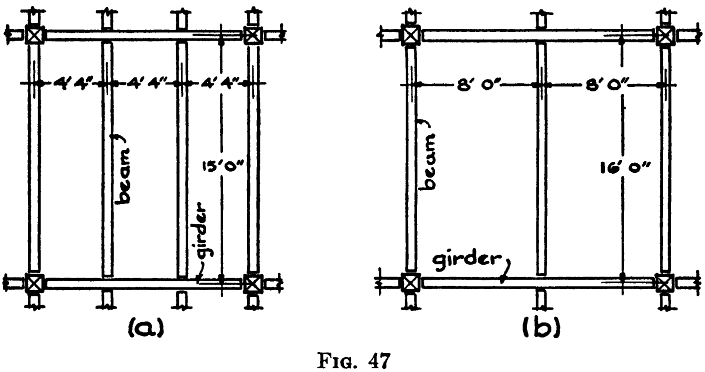


FIG. 47

In practice the actual or dressed sizes should always be used in computations. In designing beams it is essential that they be secured against lateral bending. Beams, such as joists for instance, having small breadths as compared with their depths, are held in upright positions by means of bridging. This not only prevents buckling but also helps to distribute concentrated loads to adjacent beams.

In the previous examples the beam and girder were assumed to have lengths of 14′ and 10′ respectively. Actually these distances

represent the spacing of the columns on centers and the beams have slightly shorter lengths. This, however, is common practice, and the error is on the side of safety.

PROBLEMS

49-A. Design the beam for the floor panel indicated in Fig. 47 (*a*), the floor construction to consist of 3″ plank under floor and ⅞″ top flooring. Assume the live load to be 120#/ft^2 and the timber Southern yellow pine, No. 1 Structural grade.

49-B. Design the girder shown in Fig. 47 (*a*), the data to be the same as noted in Prob. 49-A.

49-C. For the floor panel shown in Fig. 47 (*b*), the live load is 100#/ft^2 and the flooring consists of 4″ planking and a ⅞″ finished floor. Design the beams, using longleaf Southern pine, No. 1 Structural grade.

49-D. Design the girders shown in Fig. 47 (*b*), using the same data given in Prob. 49-C.

50. **Joist Tables.** Joists are the small beams spaced closely together which support floor loads. Their design consists in computing the load they support and applying the principles explained in Art. 49. In order to facilitate the selection of proper sizes many tables have been computed. Live loads of 40#ft^2 and 50#/ft^2 are usual specifications of building codes, and Tables 16 and 17 give the maximum spans for various sizes and stresses. The loads used in computing the maximum spans with respect to bending are the live load, weight of the joists, a double thickness of flooring, and a 10# allowance for a plastered ceiling. The limiting spans for deflection are computed by ignoring the plastered ceiling, the assumption being that the initial deflection from the dead load occurs before the plaster sets. You will note on examining the tables that the maximum span may be determined by either bending or deflection, and in selecting sizes of joists both of these factors should be considered.

For example, consider 3″ x 12″ joists having 900#/in^2 for an allowable extreme fiber stress in bending and whose modulus of elasticity is 1,000,000#/in^2, that is, $f = 900$ and $E = 1{,}000{,}000$. For a live load of 40#/ft^2 (Table 16) we find that with 16″ spacing the maximum span for bending is 20′ 7″, but for deflection (that is, if the deflection is not to exceed $1/360$ of the span) the maximum span is only 19′ 4″. If the building were a residence where deflec-

Table 16. Floor Joists
Live Load 40#/ft²

Maximum Allowable Lengths between Supports

Having determined from the building code or other proper authority the allowable modulus of elasticity E (in case the span is to be limited by deflection) or the allowable extreme fiber stress in bending f (in case the span is to be determined by bending) for the species and grade of lumber used, refer to the column below with the corresponding value to determine the maximum safe span for the size and spacing of joist desired. In selecting the span for deflection it should be checked with the spans for bending to see it does not exceed the length permitted for the bending stress f of the material used.

Size of Joists (Nominal) in Inches	Spacing of Joists Center to Center in Inches	Span Limited by Deflection					Span Determined by Bending										
		E =	1,000,000	1,200,000	1,400,000	1,600,000	f =	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700	1,800
2 x 6	12	L	9′ 1″	9′ 8″	10′ 2″	10′ 8″	L	9′ 6″	10′ 0″	10′ 5″	10′11″	11′ 4″	11′ 9″	12′ 3″	12′ 7″	13′ 0″	13′4″
	16	L	8 4	8 10	9 3	9 8	L	8 3	8 8	9 1	9 6	9 11	10 3	10 8	11 0	11 4	11 8
	24	L	7 3	7 9	8 2	8 6	L	6 9	7 2	7 6	7 10	8 1	8 5	8 9	9 0	9 3	9 7
2 x 8	12	L	12 1	12 10	13 6	14 1	L	12 6	13 2	13 10	14 5	15 0	15 7	16 2	16 8	17 2	17 8
	16	L	11 0	11 8	12 4	12 11	L	10 11	11 6	12 1	12 7	13 1	13 7	14 1	14 7	15 0	15 5
	24	L	9 8	10 3	10 10	11 4	L	9 0	9 6	9 11	10 4	10 9	11 2	11 7	12 0	12 4	12 8
2 x 10	12	L	15 2	16 1	17 0	17 9	L	15 9	16 7	17 5	18 2	18 11	19 7	20 4	21 0	21 7	22 3
	16	L	13 11	14 9	15 6	16 3	L	13 9	14 6	15 2	15 10	16 6	17 2	17 9	18 4	18 11	19 5
	24	L	12 3	13 0	13 8	14 3	L	11 4	11 11	12 6	13 1	13 7	14 2	14 7	15 1	15 7	16 0
2 x 12	12	L	18 4	19 5	20 5	21 4	L	18 11	19 11	20 11	21 10	22 9	23 7	24 5	25 2	26 0	26 9
	16	L	16 9	17 9	18 9	19 7	L	16 7	17 5	18 3	19 1	19 11	20 8	21 4	22 1	22 9	23 5
	24	L	14 9	15 8	16 6	17 3	L	13 8	14 5	15 1	15 9	16 5	17 0	17 8	18 3	18 9	19 4

2 x 14	12	*L*	21 4	22 7	23 10	24 11	*L*	22 0	23 3	24 4	25 5	26 6	27 6	28 5	29 4	30 3	
	16	*L*	19 7	20 9	21 10	22 10	*L*	19 4	20 4	21 4	22 4	23 3	24 1	24 11	25 9	26 6	27 4
	24	*L*	17 3	18 4	19 4	20 2	*L*	16 0	16 10	17 8	18 5	19 2	19 11	20 7	21 4	21 11	22 7
3 x 6	12	*L*	10 7	11 3	11 10	12 4	*L*	11 10	12 6	13 1	13 8	14 3	14 9	15 4	15 10	16 3	16 9
	16	*L*	9 8	10 3	10 10	11 3	*L*	10 4	10 11	11 5	12 0	12 5	12 11	13 4	13 10	14 3	14 8
	24	*L*	8 6	9 0	9 6	9 11	*L*	8 6	9 0	9 5	9 10	10 3	10 8	11 0	11 4	11 9	12 1
3 x 8	12	*L*	13 11	14 10	15 7	16 4	*L*	15 7	16 6	17 3	18 0	18 9	19 6	20 2	20 10	21 6	22 1
	16	*L*	12 9	13 7	14 4	14 11	*L*	13 8	14 5	15 2	15 10	16 5	17 1	17 8	18 3	18 10	19 4
	24	*L*	11 3	12 0	12 7	13 2	*L*	11 4	11 11	12 6	13 1	13 7	14 1	14 7	15 1	15 6	16 0
3 x 10	12	*L*	17 5	18 7	19 7	20 6	*L*	19 7	20 7	21 8	22 7	23 6	24 5	25 3	26 1	26 11	27 8
	16	*L*	16 1	17 1	18 0	18 10	*L*	17 2	18 1	19 0	19 10	20 8	21 5	22 2	22 11	23 7	24 4
	24	*L*	14 3	15 1	15 11	16 7	*L*	14 3	15 0	15 9	16 5	17 1	17 9	18 5	19 0	19 7	20 2
3 x 12	12	*L*	21 0	22 3	23 6	24 6	*L*	23 5	24 8	25 10	27 0	28 1	29 2	30 2			
	16	*L*	19 4	20 6	21 7	22 7	*L*	20 7	21 9	22 9	23 10	24 9	25 9	26 7	27 6	28 4	29 2
	24	*L*	17 1	18 2	19 2	20 0	*L*	17 1	18 0	18 11	19 9	20 7	21 4	22 1	22 10	23 6	24 3
3 x 14	12	*L*	24 5	25 11	27 4	28 7	*L*	27 2	28 8	30 0							
	16	*L*	22 6	23 11	25 2	26 4	*L*	24 0	25 3	26 6	27 8	28 10	29 11	31 0			
	24	*L*	20 0	21 3	22 4	23 4	*L*	20 0	21 1	22 1	23 1	24 0	24 11	25 10	26 8	27 6	28 3

NOTE: The above span lengths are based on

When limited by deflection,
- Maximum allowable deflection of 1/360 of the span length.
- Modulus of elasticity as noted for *E*.
- Dead load—weight of joist.
  - double thickness of flooring (5#).
  - weight of plaster ceiling ignored.
- Live load—40#/ft² with plastered ceiling.

When limited by bending strength of piece,
- Allowable stress in extreme fiber in bending as noted for *f*.
- Dead load—weight of joist.
  - double thickness of flooring (5#).
  - plastered ceiling (10#).
- Live load—40#/ft² with plastered ceiling, and 50#/ft² with unplastered ceiling.

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Table 17. Floor Joists

Live Load 50#/ft^2

Size	Spacing	Span Limited by Deflection					Span Determined by Bending										
		E =	1,000,000	1,200,000	1,400,000	1,600,000	f =	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700	1,800
2 x 6	12	L	8′ 6″	9′ 1″	9′ 6″	10′ 0″	L	8′ 9″	9′ 2″	9′ 8″	10′ 1″	10′ 6″	10′11″	11′ 3″	11′ 8″	12′ 0″	12′ 4″
	16	L	7 9	8 3	8 8	9 1	L	7 7	8 0	8 5	8 9	9 2	9 6	9 10	10 1	10 5	10 9
	24	L	6 10	7 3	7 8	8 0	L	6 3	6 7	6 11	7 2	7 6	7 9	8 0	8 4	8 7	8 10
2 x 8	12	L	11 4	12 0	12 8	13 3	L	11 7	12 2	12 9	13 4	13 11	14 5	14 11	15 5	15 11	16 4
	16	L	10 4	11 0	11 7	12 1	L	10 1	10 7	11 2	11 8	12 1	12 7	13 0	13 5	13 10	14 3
	24	L	9 1	9 8	10 2	10 7	L	8 3	8 9	9 2	9 7	9 11	10 4	10 8	11 0	11 4	11 8
2 x 10	12	L	14 3	15 2	15 11	16 8	L	14 7	15 4	16 1	16 10	17 6	18 2	18 9	19 5	20 0	20 7
	16	L	13 0	13 10	14 7	15 3	L	12 8	13 5	14 0	14 8	15 3	15 10	16 5	16 11	17 5	17 11
	24	L	11 6	12 2	12 10	13 5	L	10 5	11 0	11 7	12 1	12 7	13 0	13 6	13 11	14 4	14 9
2 x 12	12	L	17 2	18 3	19 3	20 1	L	17 6	18 5	19 4	20 2	21 0	21 10	22 7	23 4	24 1	24 9
	16	L	15 9	16 9	17 7	18 5	L	15 3	16 1	16 11	17 8	18 5	19 1	19 9	20 5	21 0	21 8
	24	L	13 10	14 8	15 6	16 2	L	12 7	13 3	13 11	14 7	15 2	15 9	16 3	16 10	17 4	17 10
2 x 14	12	L	20 1	21 4	22 5	23 6	L	20 5	21 6	22 7	23 7	24 6	25 5	26 4	27 3	28 1	28 10
	16	L	18 5	19 6	20 7	21 6	L	17 10	18 10	19 9	20 8	21 6	22 3	23 1	23 10	24 7	25 3
	24	L	16 2	17 3	18 1	18 11	L	14 9	15 7	16 4	17 0	17 9	18 5	19 0	19 8	20 3	20 10

3 x 6	12	*L*	9 11	10 6	11 1	11 7	*L*	11 0	11 7	12 1	12 8	13 2	13 8	14 2	14 7	15 1	15 6
	16	*L*	9 1	9 8	10 2	10 7	*L*	9 7	10 1	10 7	11 0	11 6	11 11	12 4	12 9	13 2	13 6
	24	*L*	8 0	8 6	8 11	9 4	*L*	7 10	8 3	8 8	9 1	9 6	9 10	10 2	10 6	10 10	11 2
3 x 8	12	*L*	13 1	13 11	14 8	15 4	*L*	14 6	15 3	16 0	16 9	17 5	18 1	18 8	19 4	19 11	20 6
	16	*L*	12 0	12 9	13 5	14 1	*L*	12 8	13 4	14 0	14 7	15 3	15 9	16 4	16 10	17 5	17 11
	24	*L*	10 7	11 3	11 10	12 4	*L*	10 5	11 0	11 6	12 1	12 7	13 0	13 6	13 11	14 4	14 9
3 x 10	12	*L*	16 6	17 6	18 5	19 3	*L*	18 2	19 2	20 1	21 0	21 10	22 8	23 5	24 2	24 11	25 8
	16	*L*	15 2	16 1	16 11	17 8	*L*	15 11	16 9	17 7	18 4	19 1	19 10	20 6	21 3	21 10	22 6
	24	*L*	13 4	14 2	14 11	15 7	*L*	13 2	13 10	14 6	15 2	15 10	16 5	17 0	17 6	18 1	18 7
3 x 12	12	*L*	19 10	21 1	22 2	23 2	*L*	21 9	22 11	24 0	25 1	26 2	27 1	28 1	29 0	29 11	30 9
	16	*L*	18 2	19 4	20 4	21 3	*L*	19 1	20 2	21 1	22 1	23 0	23 10	24 8	25 6	26 3	27 0
	24	*L*	16 1	17 1	18 0	18 10	*L*	15 10	16 8	17 6	18 3	19 0	19 9	20 5	21 1	21 9	22 5
3 x 14	12	*L*	23 1	24 6	25 10	27 0	*L*	25 3	26 8	28 0	29 2	30 5					
	16	*L*	21 3	22 7	23 9	24 10	*L*	22 3	23 6	24 7	25 9	26 9	27 9	28 9	29 8	30 7	
	24	*L*	18 10	20 0	21 0	22 0	*L*	18 6	19 6	20 5	21 4	22 3	23 1	23 11	24 8	25 5	26 2

NOTE: The above span lengths are based on

When limited by deflection,
Maximum allowable deflection of 1/360 of the span length.
Modulus of elasticity as noted for *E*.
Dead load—weight of joist.
double thickness of flooring (5#).
weight of plaster ceiling ignored.
Live load—50#/ft^2 with plastered ceiling.

When limited by bending strength of piece,
Allowable stress in extreme fiber in bending as noted for *f*.
Dead load—weight of joist.
double thickness of flooring (5#).
plastered ceiling (10#).
Live load—50#/ft^2 with plastered ceiling, and
60#/ft^2 with unplastered ceiling.

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tion was a matter of importance, the limiting span for these joists would be 19′ 4″.

51. **Design of Joists.** In practice, joist sizes are invariably selected by referring to tables. For special cases, such as joists supporting partitions in addition to floor loads, it is necessary to compute the required section modulus and to select the section accordingly. The following two examples will illustrate the methods employed.

EXAMPLES

Example. Joists of what size should be used to support a live load of 40#/ft² (if the joists are spaced 16″ on centers and the span is 22′ 0″)? The joists will support a double thickness of flooring and a plastered ceiling. The timber is to be the 1,200# f grade of Eastern spruce.

Solution. Reference to Table 3 shows that $f = 1{,}200\#/\text{in}^2$ and $E = 1{,}200{,}000\#/\text{in}^2$.

Computing the loads,

Live load	=	40.0
Double thickness of floor	=	5.0
Weight of joist (estimated)	=	8.8
Plastered ceiling	=	10.0
		63.8#/ft²

Since the joists are to be 16″ on centers, the load per linear foot on one joist will be $63.8 \times 16/12$ or 85#. The total load on one joist will be 85×22 or 1,870#.

$$M = \frac{Wl}{8} = \frac{1{,}870 \times 22 \times 12}{8} = 61{,}710''\#$$

$$\frac{M}{f} = S = \frac{61{,}710}{1{,}200} = 51.3\text{in}^3$$

the required section modulus. By referring to Table 12 we find that a 3″ x 12″ has a section modulus of 57.86in³ and therefore this section is acceptable for strength in bending.

Now let us investigate the deflection. As stated in Art. 50, the initial deflection from the dead load occurs before the plaster sets, hence the weight of the plaster ceiling may be omitted in determin-

ing the weight used in computing the deflection. The floor load computed above is 63.8#/ft^2, and this includes the 10#/ft^2 allowance for plaster. Thus 63.8 − 10 = 53.8#/ft^2, the load to be used in computing the deflection.

The joists being 16″ on centers, 53.8 × 16/12 = 71.9#, the load per linear foot on one joist. Thus the total load on one joist is 71.9 × 22 or 1,580#.

The length of span is 22′ 0″ or 264″. From Table 12 we find that the moment of inertia I of a 3″ x 12″ section is 332.69in^4.

$$D = \frac{5}{384} \times \frac{Wl^3}{EI} \quad \text{Table 4}$$

Then

$$D = \frac{5}{384} \times \frac{1{,}580 \times 264^3}{332.69 \times 1{,}200{,}000} = 0.94''$$

the actual deflection.

The allowable deflection is $\frac{22 \times 12}{360} = 0.73''$. Thus it is seen that the actual deflection exceeds the allowable, and the 3″ x 12″ joists with 16″ spacing will deflect excessively.

Floor joists of this type are invariably amply strong with respect to horizontal shear and need not be tested.

The above computations show the method that might be employed, but since tables have been compiled by use of the same computations why not employ them? Without any computations the following steps are all that is necessary.

Refer to Table 16. This table gives us the maximum spans for joists having live loads of 40#/ft^2. Here we find in the column "f = 1,200," that 3″ x 12″ joists 16″ on centers must not have spans in excess of 23′ 10″. This, of course, is the maximum span as determined by strength in bending.

Now let us see about the deflection of these joists. Under the column "E = 1,200,000," we note that 3″ x 12″ joists 16″ on centers may have a span not to exceed 20′ 6″ if the deflection is not to be excessive. But our problem calls for a span of 22′ 0″. The table shows, without computations from us, that the 3″ x 12″ joists 16″ on centers are adequate with respect to strength in bending but that the deflection would be excessive. This is

exactly the same information found by our computations above, but see what time may be saved by use of the table!

By use of the table we find that 3″ x 14″ joists 16″ on centers are acceptable for this particular problem. Still another choice would be 3″ x 12″ joists 12″ on centers.

EXAMPLE. Floor joists spaced 16″ on centers, having a span of 14′, support a live floor load of 40#/ft^2 and in addition a plastered partition 9′ in height at 4′ from an end support, Fig. 48 (*a*). The

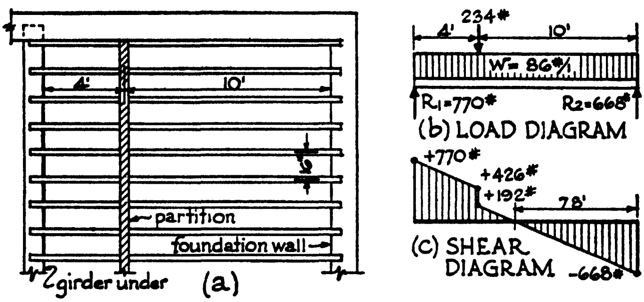


FIG. 48

joists support double flooring and a plastered ceiling underneath. Design the floor joists using Select Structural grade of Eastern hemlock.

SOLUTION. $f = 1{,}100\#/\text{in}^2$ and $E = 1{,}100{,}000\#/\text{in}^2$ (Table 3).

Floor Loads

Live load	= 40.
Top floor	= 2.5
Under floor	= 2.5
Plaster ceiling	= 10.
Joist (assumed)	= 10.
Total	= 65.0#/ft^2

Joist spacing, 16″ o.c. Therefore $65 \times 16/12 = 86$ pounds per linear foot, the load on one joist.

Load due to partition, assuming 20#/ft^2 (Table 13), $1.3 \times 9 = 11.7$ft^2 of partition supported by one joist (joist 16″ o.c.); $11.7 \times 20 = 234.0\#$, partition load on one joist.

Select the joist for bending: In order to find the position of the maximum bending moment we must draw the shear diagram.

The load diagram is shown in Fig. 48 (*b*). R_1 and R_2 are com-

puted as shown in Art. 24 and are found to be 770# and 668# respectively. The shear diagram, Fig. 48 (c), is next drawn (see Art. 28), and it is observed that the shear passes through zero at a point between the concentrated load and the right reaction. This is the location of the maximum bending moment. To simplify mathematics consider the forces to the right, then $V = 0 = 668 - (86 \times x)$ or $x = 7.8'$, the point of maximum bending moment. Writing the value of M at this point, again considering the forces to the right of the section,

$$M = (668 \times 7.8) - \left(86 \times 7.8 \times \frac{7.8}{2}\right) = 2{,}594'\# = 31{,}128''\#$$

$$\frac{M}{f} = S = \frac{31{,}128}{1{,}100} = 28.3\text{in}^3, \text{ the required section modulus}$$

Table 12 shows that 2″ x 12″ joists have a section modulus of 35.82in^3 and are therefore acceptable for bending.

Test the joist for deflection:

$$\text{Allowable deflection} = \frac{14 \times 12}{360} = 0.46''$$

Find the distributed load that produces the same bending moment, 2,594′# (see second example under Art. 49),

$$M = \frac{Wl}{8}$$

$$2{,}594 = \frac{W \times 14}{8}$$

$$W = 1{,}482\#$$

$$\text{Actual deflection } D = \frac{5Wl^3}{384EI}, \qquad I = 206 \quad \text{(Table 12)}$$

$$D = \frac{5 \times 1{,}482 \times (14 \times 12)^3}{384 \times 1{,}100{,}000 \times 206} = 0.40''$$

Since the actual deflection is less than the allowable, the selection, 2″ x 12″ joists 16″ on centers, is acceptable.

PROBLEMS

51-A. Compute the size of joists required for a live load of 50#/ft^2 of typical floor construction for a span of 16′. Joists to be 1,000# *f* grade of Eastern spruce, spaced 16″ o.c.

51-B. Check the sizes computed in Prob. 51-A by Table 17.

51-C. Design floor joists having a span of 16′, a live load of 40#/ft^2, and a plastered partition 10′ in height placed at 6′ from one end of the joist, timber to be 1,000# *f* grade of spruce.

51-D. To what span are 3″ x 12″ joists 16″ on centers limited when they carry a 50#/ft^2 live load, double flooring, and a plastered ceiling? The timber is 1,200# *f* Eastern spruce.

51-E. Floor joists having a span of 12′ 0″ have a live load of 40#/ft^2, double flooring, and a plastered ceiling. In addition, the joists support at the center of the span a plastered stud partition 9′ 0″ in height. Design the joists, using the Select Structural grade of Eastern hemlock.

52. **Plank or Mill Floors.** In mill construction, planks 2″ or more in thickness, laid on the flat, span from beam to beam. Joists are not used in this type of construction, the beams being spaced 5′ to 12′ on centers, depending on the load and the kind of timber used. Since the plank is dressed on both sides to obtain a uniform thickness, there is a discrepancy between the nominal and actual thickness. These thicknesses together with other properties are given in Table 18.

Table 18. Properties of Yellow Pine Mill Floors *

Nominal Thickness	Actual Thickness	Breadth	Area of Section	Weight per Square Foot	Moment of Inertia	Section Modulus
In	In	In	In^2	Lb	In^4	In^3
2	1⅝	12	19.5	5.40	4.29	5.28
2½	2⅛	12	25.5	6.90	9.59	9.03
3	2⅝	12	31.5	8.70	18.09	13.78
4	3⅝	12	43.5	12.00	47.63	26.28
5	4⅝	12	55.5	15.40	98.93	42.78
6	5½	12	66.0	18.30	166.37	60.50

* Courtesy of Southern Pine Association, New Orleans, La.

In computing the thickness of the plank, a width of 12″ is assumed and the methods employed in the design of beams are

used. Note that Table 18, in giving the properties of plank floors, assumes a 12″ width.

Although the computations involved in designing a plank floor are relatively simple, the proper thickness may be found directly by the use of Table 19. In this table allowable spans are given for various live loads for thicknesses of 2½″, 3″, 4″ and 5″. The lines marked "deflection" give the maximum spans for these four thicknesses, beyond which the deflection exceeds 1/360 of the span.

Table 19. Maximum Spans for Yellow Pine Mill Floors in Feet and Inches*

$E = 1{,}600{,}000\#/in^2$

Nominal Thickness	Actual Thickness	Fiber Stress	Span in Feet					
			Live load in pounds per square foot					
In	In	Pounds per square inch	50#	100#	125#	150#	175#	200#
2½	2⅛	1,200	11– 3	8– 2	7– 5	6– 9	6– 4	5–11
		1,300	11– 9	8– 7	7– 9	7– 1	6– 7	6– 2
		1,500	12– 7	9– 2	8– 3	7– 7	7– 0	6– 7
		1,600	13– 0	9– 6	8– 6	7–10	7– 3	6–10
		1,800	13–10	10– 1	9– 1	8– 4	7– 8	7– 3
		Deflection	7– 4½	5–11½	5– 7	5– 3	5– 0	4– 9½
3	2⅝	1,200	13– 8	10– 1	9– 1	8– 4	7– 9	7– 3
		1,300	14– 3	10– 6	9– 6	8– 8	8– 1	7– 7
		1,500	15– 4	11– 3	10– 2	9– 4	8– 8	8– 2
		1,600	15–10	11– 8	10– 6	9– 7	8–11	8– 4
		1,800	16– 9	12– 4	11– 2	10– 3	9– 6	8–11
		Deflection	9– 0	7– 4	6–10	6– 5½	6– 1½	5–10½
4	3⅝	1,200	18– 5	13– 8	12– 4	11– 5	10– 7	10– 0
		1,300	19– 2	14– 3	12–11	11–10	11– 0	10– 4
		1,500	20– 7	15– 4	13–10	12– 9	11–10	11– 2
		1,600	21– 3	15–10	14– 4	13– 2	12– 3	11– 6
		1,800	22– 7	16– 9	15– 2	13–11	13– 0	12– 2
		Deflection	12– 2½	10– 0	9– 4½	8–10½	8– 5½	8– 1½
5	4⅝	1,200	22–10	17– 8	15– 7	14– 5	13– 5	12– 7
		1,300	23–10	17–11	16– 3	14–11	13–11	13– 1
		1,500	25– 7	19– 3	17– 5	16– 1	15– 0	14– 1
		1,600	26– 5	19–11	18– 0	16– 7	15– 6	14– 7
		1,800	28– 0	21– 1	19– 1	17– 7	16– 5	15– 5
		Deflection	15– 3½	12– 8	11–10½	11– 2½	10– 8½	10– 3½

* Courtesy of Southern Pine Association, New Orleans, La.

EXAMPLE

Example. Compute the thickness of a Southern yellow pine plank floor, $f = 1{,}200\#/in^2$ grade, having beams 8′ on centers and a live load of $125\#/ft^2$.

SOLUTION.

Loads		
Live load	=	125
Top flooring	=	3
3″ plank (assumed)	=	10
Total	=	138#/ft²

Consider the flooring as a beam 12″ wide having a span of 8′. Then $w = 138\#$, the load per linear foot.

$$M = \frac{wl^2}{8} = \frac{138 \times 8 \times 8}{8} = 1{,}104'\# = 13{,}248''\#$$

$$\frac{M}{f} = S = \frac{13{,}248}{1{,}200} = 11.04\text{in}^3$$

Table 18 shows that planking 12″ wide with a nominal thickness of 3″ has a section modulus of 13.78in³, and this thickness is ample for strength. However, when the floor is tested for deflection this thickness is found to be inadequate, and the next thickness, 4″, will be required.

Actually, computations are not required. The correct thickness of flooring can be found directly by using Table 19. Under the column headed 125#, a 3″ floor having $f = 1{,}200\#/\text{in}^2$, we find the maximum span to be 9′ 1″, but we note also that the maximum deflection for 3″ thickness is 6′ 10″. For a 4″ thickness the maximum span for deflection is 9′ 4½″. Therefore this size is acceptable.

PROBLEMS

52-A. Compute the thickness of plank flooring for a floor having a live load of 175#/ft² with beams 10′ o.c., the timber to be Southern yellow pine of Structural Square Edge and Sound grade. Check results by Table 19.

52-B. What thickness of yellow pine plank floor should be used for a beam spacing of 10′ if the live load is 125#/ft² and $f = 1{,}500\#/\text{in}^2$?

52-C. What live load will a 4″ Southern yellow pine, No. 1 Structural grade plank floor support, the deflection not to be excessive, if the beams are spaced 10′ 0″ apart?

52-D. What thickness of Southern yellow pine plank floor should be used for a live load of 50#/ft² and a beam spacing of 12′ 0″?

52-E. What live load will a 3″ prime Structural longleaf Southern pine plank floor support if the beam spacing is 9′ 0″?

52-F. What thickness of Southern yellow pine planking should be used for a beam spacing of 7′ 6″ if the live load is 175#/ft² and f = 1,300#/in²?

53. **Timber Purlins.** A roof purlin is the beam which spans from truss to truss and transfers the roof loads to the panel points. For convenience in framing, the purlins are often placed on or framed into the truss so that one major axis of the purlin takes the same pitch as the roof, Fig. 49 (*a*). The dead and snow loads, however, are vertical, not parallel to the major axes of the purlin.

Suppose the load on the purlin to be represented by the line P, Fig. 49 (*b*). The two components of P, one normal to the pitch of the roof and the other parallel to it, are represented graphically as V and H. We may assume that the tendency of the purlin to bend in the direction of H is resisted by the roofing material, and the problem which results is to design the purlin to resist bending due to V, parallel to a major axis.

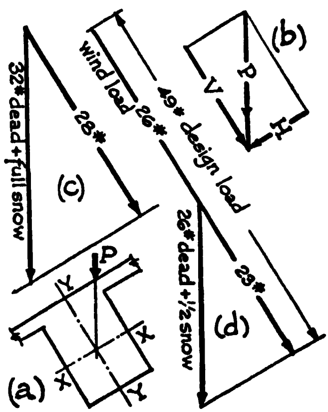


FIG. 49

The loads on roofs are the dead, snow, and wind, the dead and snow being vertical and the wind load being assumed to be normal to the surface of the roof. It is improbable that all three will act at a maximum magnitude at one time, the probable combinations being: first, maximum dead and snow loads; second, maximum dead and wind and ½ the maximum snow loads. Both combinations are computed, and the maximum force normal to the pitch of the roof is accepted as the load to be used in designing the purlin.

EXAMPLE

EXAMPLE. Consider a roof having trusses spaced 14′ on centers and purlins 6′ apart. The roof has a pitch of 30°, and the roofing material is slate on 2″ planking. If the snow load is assumed to be 12#/ft² and the wind load 26#/ft², compute the dimensions of the purlins, using the f = 1,200#/in² grade of Southern yellow pine.

SOLUTION.

Dead load	
Slate	= 15
Plank	= 5
Total dead load	= 20#/ft^2

First Combination		Second Combination	
Dead load	= 20	Dead load	= 20
Full snow	= 12	½ snow load	= 6
Total	= 32#/ft^2	Total	= 26#/ft^2 + wind load

Lay off 32# vertically to a convenient scale, Fig. 49 (*c*). The component of 32# normal to the pitch of the roof is found by scaling to be 28#. This is the first combination. The normal component of 26# is 23#, Fig. 49 (*d*); 23# added to 26#, the wind load, is 49#, Fig. 49 (*d*). This is the second combination and will be used as the design load in the computations.

Data: f = 1,200#/in^2 and E = 1,600,000#/in^2.

Loads	
Load per linear foot = 6 × 49	= 294#
Purlin (estimated)	= 20
Total	= 314#/ft, the design load

$$\frac{M}{f} = S = \frac{314 \times 14 \times 14 \times 12}{8 \times 1{,}200} = 76.9\text{in}^3, \text{ the required section modulus}$$

Referring to Table 12, a 6″ x 10″ has a section modulus of 82.73in^3. This size is acceptable provided that the deflection is not excessive.

$$\text{Allowable deflection} = \frac{14 \times 12}{360} = 0.46''$$

Actual deflection,

$$D = \frac{5Wl^3}{384EI} = \frac{5 \times 314 \times 14 \times (14 \times 12)^3}{384 \times 1{,}600{,}000 \times 392.96} = 0.43''$$

Accept 6″ x 10″ purlins.

PROBLEMS

53-A. Design roof purlins in accordance with the following data: roof trusses 15′ o.c., purlins 7′ apart, tin roof, 3″ plank, snow load 10#/ft^2, wind load 24#/ft^2, roof pitch 35°, timber Southern yellow pine f = 1,200#/in^2 grade.

53-B. Design the purlins for a roof having trusses spaced 12′ 0″ o.c., the purlins being spaced 5′ 0″ apart. The roof has a 45° pitch and the roof construction is Spanish roofing tile on 2″ planking. The snow load is 10#/ft^2 and the wind load is 36#/ft^2. Purlin timber is to be Douglas fir, Select Structural grade.

54. **Timber Columns.** Timber columns are generally square or rectangular in cross section. As applied to timber columns, the term *slenderness ratio* is $\frac{l}{d}$, the length in inches divided by *the least lateral dimension.* Columns having a slenderness ratio less than 11 are called *short columns,* and the full allowable compressive strength of the material is used. Building codes differ concerning the maximum permissible value of $\frac{l}{d}$, but 30 is a fair average.

Owing to the tendency of a column to bend, the greater the length the greater must be the dimensions of the cross section. A short column will support a load equal to the area of the cross section multiplied by the allowable compressive strength of the material, but if the length increases the stress per square inch must be smaller and the area of the cross section larger. There are many formulas for finding the permissible unit stress for timber columns, one of the simplest being

$$f = C\left(1 - \frac{l}{80d}\right)$$

in which f = the maximum permissible unit stress, in pounds per square inch for the column cross section

C = the maximum allowable unit stress parallel to the grain for short blocks, in pounds per square inch (Table 3)

l = the unsupported length of the column, in inches

d = the least width or diameter of the cross section, in inches.

Timber columns may be designed readily by use of the above formula, as will be shown, but Tables 20 and 21 have been com-

puted by means of the above formula and the proper size may be found at a glance. The values of C in the tables corresponding to various species are found in certain building codes. They are con-

Table 20. Safe Axial Load in Kips for Square Timber Columns *

$$f = C\left(1 - \frac{l}{80d}\right)$$

NOTE: Safe load on round columns is 0.785 times the safe load on square columns of the same diameter.

Nominal Size In	Actual Size In	Area In²	Length, Feet	Douglas Fir and Longleaf Yellow Pine $C = 1{,}100$	Oak $C = 900$	Short-leaf Yellow Pine $C = 800$	Norway and White Pine $C = 700$	Hem-lock $C = 500$
6 x 6	5½ x 5½	30.3	8	26.0	21.2	18.9	16.5	11.8
			10	24.2	19.8	17.6	15.4	11.0
			12	22.4	18.4	16.3	14.3	10.2
8 x 8	7½ x 7½	56.3	8	52.0	42.5	37 8	33.0	23 6
			10	49.5	40.5	36.0	31.5	22.5
			12	47.0	38.4	34.2	29.9	21.4
			14	44.5	36.4	32.4	28.4	20.2
			16	42.0	34.4	30.6	26.8	19.1
			18	39.6	32.4	28.8	25.2	18.0
10 x 10	9½ x 9½	90.3	8	86.8	71.0	63.1	55 2	39 4
			10	83.6	68 4	60.8	53.2	38.0
			12	80.1	65.5	58.3	51.0	36.4
			14	77.2	63.3	56.2	49.2	35.1
			16	74.3	60.8	54.0	47.3	33.8
			18	71.0	58 1	51.6	45.2	32.2
			20	67.8	55.5	49.4	43.2	30.9
12 x 12	11½ x 11½	132.3	8	130.5	106.9	95.0	83.1	59.4
			10	127.0	103.8	92.2	80.7	57.6
			12	123 0	100.7	89.4	78.2	55 9
			14	119.0	97.4	86.5	75.7	54.1
			16	115.2	94.4	83.9	73.4	52.4
			18	111.3	91.1	81.0	70.9	50.6
			20	109.5	89.6	79.6	69.7	49.8
14 x 14	13½ x 13½	182.3	8	183	149.8	133.0	116.3	83.1
			10	178	145.9	129.7	113.3	81.0
			12	174	142.1	126.3	110.6	79.0
			14	169	138.5	123.0	107.8	76.9
			16	165	135.0	120.0	105.0	75.0
			18	161	131.5	116.9	102.2	73.0
			20	156	127.8	113.7	99.4	71.0

* Taken from Kidder-Parker, "Architects' and Builders' Handbook."

servative and will be used in the following examples and problems for purposes of illustration. In practice the designer must con-

form to the formulas and unit stresses required by his local building code. The loads in the tables are in kips, that is, units of 1,000#.

Table. 21. Safe Axial Load in Kips for Rectangular Timber Columns*

$$f = C\left(1 - \frac{l}{80d}\right)$$

Nominal Size	Actual Size	Area	Length, Feet	Douglas Fir and Longleaf Yellow Pine	Oak	Short-leaf Yellow Pine	Norway and White Pine	Hem-lock
In	In	In^2		$C=1{,}100$	$C=900$	$C=800$	$C=700$	$C=500$
6 x 8	5½ x 7½	41.3	8	35 4	29.0	25.8	22.6	16.1
			10	33.0	27.0	24.0	21.0	15.0
			12	30.5	25.0	22.2	19.4	13.9
			14	28.1	23.0	20.4	17.9	12.8
6 x 10	5½ x 9½	52.3	8	45.0	36.7	32 6	28.6	20.4
			10	41.9	34.2	30.4	26.6	19.0
			12	38.6	31.6	28.1	24.6	17.5
			14	35.5	29.0	25.8	22.6	16.1
6 x 12	5½ x 11½	63.3	8	54.4	44.5	39.5	34.6	24.7
			10	50.6	41.4	36.8	32.2	23.0
			12	46.8	38 2	34.0	29.8	21.2
			14	43.0	35.2	31.2	27.4	19.5
8 x 10	7½ x 9½	71.3	8	65.9	53.9	47.9	41.9	29.9
			10	62 8	51.3	45.6	39.9	28.5
			12	59.5	48 7	43.3	37.9	27.0
			14	56.5	46.2	41 0	35.9	25.6
			16	53.3	43.6	38.8	33.9	24.2
			18	50 2	41.0	36.5	31.9	22.8
			20	47.0	38.5	34.2	29.9	21.4
8 x 12	7½ x 11½	86.3	8	79.8	65.2	58.0	50.7	36.2
			10	76.0	62.1	55.2	48.3	34.5
			12	72.1	59.0	52.5	45.9	32.8
			14	68.4	55 9	49.7	43.5	31.0
			16	64.5	52 8	46.9	41.0	29.3
			18	60.8	49.7	44.2	38.6	27.6
			20	57.0	46 6	41.4	36.2	25.9
10 x 12	9½ x 11½	109.3	8	105.0	86.0	76.4	66.9	47.8
			10	101.2	82.9	73.6	64.5	46.0
			12	97.1	79.5	70.6	61.8	44.1
			14	93.6	76.6	68.1	59.6	42.5
			16	90.0	73.5	65.4	57.2	40.8
			18	86.0	70.4	62.5	54.7	39.1
			20	82.3	67.3	59.8	52.4	37.4

* Taken from Kidder-Parker, "Architects' and Builders' Handbook."

55. Design of Timber Columns. The design of timber columns is accomplished by the trial method; that is, a column having a

given cross section is assumed and then tested to see whether or not it meets the requirements of the problem. Data given include the length of the column, the load, and the kind of timber. Regardless of the formula used, the following steps may be employed.

STEP 1. Assume a cross-section area of approximately the proper dimensions.

STEP 2. Solve for f in the timber column formula, substituting for l the length of the column in inches and for d the least width or diameter in inches.

STEP 3. f found in Step 2 is in units of pounds per square inch. Multiply this stress by the number of square inches in the cross section of the column assumed, and the result is the allowable or safe load for the column.

STEP 4. Compare the allowable load found in Step 3 with the actual load to be supported. If the actual load is the greater, the cross section assumed has been too small. If this is true, assume a larger cross section and test it in the same manner.

EXAMPLE

EXAMPLE. Design a timber column having a length of 14′ and an axial load of 80,000#, the timber to be Southern yellow pine, $C = 1{,}100\#\text{in}^2$.

SOLUTION. STEP 1. Assume the cross section to be 10″ x 10″. This is the nominal size, the actual being 9.5″ x 9.5″ = 90.25in^2.

STEP 2. $f = C\left(1 - \dfrac{l}{80d}\right) = 1{,}100\left(1 - \dfrac{14 \times 12}{80 \times 9.5}\right) = 857\#/\text{in}^2.$

STEP 3. The allowable load on the assumed column equals 857# $\times$ 90.25 = 77,344#.

STEP 4. Since 77,344# is less than 80,000#, the assumed section is too small.

Second Trial. STEP 1. Assume a 10″ x 12″ column, actual size = 9.5″ x 11.5″.

STEP 2. $f = 1{,}100\left(1 - \dfrac{14 \times 12}{80 \times 9.5}\right) = 857\#/\text{in}^2$. Note that this value is the same as that found in the first trial section owing to the fact that d in each case is the least width or 9.5″.

STEP 3. The area of the section = $9.5 \times 11.5 = 109.3\text{in}^2$; $109.3 \times 857\# = 93{,}670\#$, the allowable or safe load.

STEP 4. Since the allowable load is larger than 80,000# the 10″ x 12″ column is acceptable.

The proper size of column for this load and length could have been selected directly from Tables 20 and 21. Note that the safe load for a 12″ x 12″ is 119,000#, and for an 8″ x 12″, 68,400#. A 10″ x 12″ is the most economical section.

PROBLEMS

55-A. Design a longleaf yellow pine column having a length of 16′ and a load of 72,000#.

55-B. What size of oak column, 8′ in length, is required for a load of 25,000#?

55-C. Design a shortleaf yellow pine round column 14′ in length to support a load of 65,000#.

55-D. A Douglas fir column 14′ in length has a load of 165,000#. What should be the dimensions of its cross section?

55-E. A 6″ x 8″ post of No. 1 Structural longleaf Southern pine is 5′ 0″ long. What is the allowable axial load?

55-F. What size of hemlock column, 12′ 0″ in length, is required to support an axial load of 27,000#?

55-G. What is the allowable axial load on a Norway pine column 6″ x 10″ in cross section and 14′ in length?

55-H. What minimum size square Douglas fir column, 12′ 0″ in length, should be used to support an axial load of 45,000#?

SECTION III

STEEL CONSTRUCTION

56. Structural Shapes. The structural steel shapes which are most commonly used in building construction are I-beams, channels, H-columns, angles, and plates. Properties of various rolled shapes are given in Tables 5 to 11 inclusive. These tables are necessarily brief; complete data may be found by referring to manufacturers' handbooks. Beams are identified by their nominal depth and the weight per linear foot. Typical examples are

10″ I 25.4#	10″ Standard I-beam, 25.4#
6″ B 12#	6″ miscellaneous beam, 12#
8″ x 8″ M 32.6#	8″ miscellaneous column, 32.6#
Pl. 18″ x 12″ x ½″	18″ x 12″ x ½″ plate
14″ WF 30#	14″ wide flange, 30#
8″ [11.5#	8″ channel, 11.5#
2—4″ x 3″ x ½″ Ls	2—4″ x 3″ x ½″ Ls

Since, with a few exceptions, the Bethlehem and Carnegie Steel companies roll beams of identical depths and weights, the letters WF are used in place of C, B, and G. WF indicates a wide flange beam to distinguish it from the American Standard beam.

57. Steel Beams. The most economical rolled section used as a beam is the I-shaped section or I-beam. It is symmetrical with respect to the two major axes and, when used as a beam, rests on a flange. Most of the material in the cross section is contained in the flanges, and for bending this condition is ideal since the greatest bending stresses occur in the flange areas.

In general, a steel beam should have a cross-section area to resist adequately bending, shear, and deflection.

58. Flexure or Bending. To design a steel beam for bending is quite simple. It consists in merely applying the flexure formula, $\frac{M}{f} = S$. First, compute the maximum bending moment, as previ-

ously explained. Second, divide the maximum bending moment by the allowable extreme fiber stress, building codes permitting either 18,000 or 20,000#/in^2. Third, the quotient obtained from the second step gives the section modulus S of the required section. Therefore refer to the tables of properties of steel sections and select a section having a section modulus equal to or greater than that required. In general, the lightest-weight section is the most economical. One of the most common errors in using the flexure formula is that of mixing denominations. That is, if f is in units of pounds per square inch M must be in inch-pounds, not foot-pounds.

It is important that the beginner understand thoroughly the procedure just described, for it is applicable to any type of loading. In practice, however, beams having uniformly distributed loads are probably the most common type, and designers may refer to tables to select directly the proper size of beam. It is necessary to know only the load and the span. See Art. 64 and Tables 23-A and 23-B.

EXAMPLES

EXAMPLE. Design an I-beam for flexure to support a uniformly distributed load of 30,000# including the weight of the beam. The beam is a simple beam having a span of 14′.

SOLUTION. The flexure formula is $\frac{M}{f} = S$. $f = 18{,}000\#/\text{in}^2$ (Table 2).

$$M = \frac{Wl}{8} \quad \text{(Table 4)}$$

$$M = \frac{30{,}000 \times 14 \times 12}{8} = 630{,}000''\#$$

$$S = \frac{M}{f} = \frac{630{,}000}{18{,}000} = 35\text{in}^3, \text{ the required section modulus}$$

Referring to Table 5 we find that a 12″ I 31.8#, has a section modulus of 36in^3, and this beam is acceptable. This is a Standard I-beam. Another acceptable beam is a 12″ WF 28# (Table 6).

Note that these beams might have been selected directly from Table 23-A without the above computations.

Example. Design for flexure a steel beam 18′ in length having a uniformly distributed load of 400 pounds per linear foot, including the weight of the beam, and two concentrated loads of 8,000# each, at 6′ from the supports.

Solution. Total load = 8,000 + 8,000 + (400 × 18) = 23,200#.

$$R_1 = R_2 = \frac{23{,}200}{2} = 11{,}600\#$$

The maximum bending moment occurs at the center of the span, or

$$M_{(x=9)} = (11{,}600 \times 9) - [(400 \times 9 \times 4.5) + (8{,}000 \times 3)]$$

$$= 64{,}200'\# = 770{,}400''\#$$

$$\frac{M}{f} = S = \frac{770{,}400}{18{,}000} = 42.8\text{in}^3, \text{ the required section modulus}$$

Referring to Tables 5 and 6, select

12″ I 40.8#

or

12″ WF 40#

or

14″ WF 34#

Note that the 14″ WF 34# is the lightest weight in this instance, but has the greatest depth. Occasionally construction limits the depth of a beam, and the selection must be made with the depth as a controlling factor.

PROBLEMS

58-A. Design for flexure a simple beam 14′ in length, having a total uniformly distributed load of 19,800#. Let $f = 18{,}000\#/\text{in}^2$.

58-B. Design for flexure a beam having a span of 16′ with a concentrated load of 12,500# at the center of the span. Let $f = 18{,}000\#/\text{in}^2$.

58-C. A beam 15′ in length has three concentrated loads of 4,000#, 5,000#, and 6,000# at 4′, 10′, and 12′ respectively from the left-hand support. If $f = 18{,}000\#/\text{in}^2$, design the beam for flexure.

58-D. A beam 30′ in length has concentrated loads of 9,000# each, and at the third points also a total uniformly distributed load of 30,000#. If f = 18,000#/in², design the beam for flexure.

58-E. Design for flexure a beam 12′ in length having a uniformly distributed load of 2,000 pounds per linear foot and a concentrated load of 8,400# at 5′ from a support. Let f = 18,000#/in².

58-F. A beam 19′ in length has concentrated loads of 6,000# and 9,000# at 5′ and 13′ respectively from the left-hand support. In addition, there is a uniformly distributed load of 1,200 pounds per linear foot beginning at 5′ from the left support and continuing to the right support. If f = 18,000#/in², design the beam for flexure.

58-G. A steel beam 16′ 0″ in length has two uniformly distributed loads, one of 200 pounds per linear foot extending 10′ 0″ from the left support and the other of 100 pounds per linear foot extending from a point 10′ 0″ from the left support to the right support. In addition there is a concentrated load of 8,000# at 10′ 0″ from the left support. Design the beam for flexure, using an allowable extreme fiber stress of 20,000#/in².

58-H. Design for flexure, using an allowable extreme fiber stress of 20,000#/in², a simple beam 12′ 0″ in length having two concentrated loads of 5,000# each, one at 4′ 0″ from the right end and the other at 4′ 0″ from the left end.

58-I. A cantilever beam 8′ 0″ long has a uniformly distributed load of 600 pounds per linear foot. If f = 20,000#/in², design the beam for flexure.

58-J. A cantilever beam 6′ 0″ in length has a concentrated load of 12,300# at its unsupported end. If f = 20,000#/in², design the beam for flexure.

59. **Shear.** After a beam is designed for flexure it should be investigated for shear. Most beams sufficiently large to resist bending or flexure are also sufficiently large for shear; consequently, this step is often omitted. However, short beams or beams with relatively large loads near the supports should always be tested for shear; occasionally the beam must be increased in size.

Referring to Art. 47, we find that a beam has a tendency to fail by shear by the fibers tending to slide past each other both vertically and horizontally. It can be shown also that at any section of a beam the total intensities of the vertical and horizontal shearing stresses are equal. In considering steel beams we are concerned principally with horizontal shear. The shearing stresses are not equally distributed over the cross section but are greatest at the neutral axis and zero at the extreme fibers. Because of this fact, *it is assumed that the web of steel beams is the only portion of the cross section resisting shear.* If the average shearing unit stress does not exceed an allowable shearing unit stress, usually 12,000 or

13,000#/in^2, the beam is safe with respect to shear. We may say, then,

$$f_s = \frac{V}{A} \qquad \text{or} \qquad V = f_s A$$

in which f_s = the shearing unit stress, in pounds per square inch
V = maximum vertical shear, in pounds
A = the area of the web, in square inches, the depth of the beam multiplied by the web thickness.

Remember that the maximum vertical shear is equal to the greater reaction for simple beams, and that for beams symmetrically loaded each reaction equals ½ the total load.

The thickness of the webs of beams is given in the tables of properties. See Tables 5 and 6.

EXAMPLES

Example. A 15″ I 42.9#, having a length of 14′ has a total uniformly distributed load of 50,000#. Investigate for shear.

Solution. Total load = 50,000#.

$$R_1 = R_2 = \frac{50,000}{2} = 25,000\#$$

Therefore the maximum vertical shear V = 25,000#.

The depth of the beam = 15″, and the web thickness = 0.41″ (Table 5). The area of the web $A = 15 \times 0.41 = 6.15$in^2.

$$f_s = \frac{V}{A} = \frac{25,000}{6.15} = 4,065\#/\text{in}^2$$

Since 4,065#/in^2 is less than 12,000#/in^2 (Table 2), the beam is amply large to resist shear.

Example. With respect to shear, what should be the maximum magnitude of a concentrated load at the center of the span of an 18″ I 54.7#?

Solution. $A = 18 \times 0.46 = 8.28$in^2 (Table 5).

Maximum allowable shearing unit stress = 12,000#/in^2 (Table 2)

Therefore, since $V = f_s \times A$, $V = 12,000 \times 8.28$, or V = 99,360#.

If V = 99,360# and the load is concentrated at the center of the span, $R_1 = R_2$ = 99,360#, and the load = 99,360 × 2 = 198,720#.

PROBLEMS

59-A. A 10″ I 30#, has a uniformly distributed load of 40,000#. Investigate for shear.

59-B. A grillage beam, 8″ I 18.4#, has a maximum shear of 32,000#. Is the beam large enough?

59-C. A 15″ I 42.9# has two concentrated loads of 60,000# at equal distances from the supports. Investigate for shear.

59-D. A simple beam composed of a 14″ WF 34#, 15′ 0″ in length, has a uniformly distributed load of 20,000# and a concentrated load of 10,000# at the center of the span. Investigate for shear.

59-E. With respect to shear, what is the maximum distributed load that may be placed on a 20″ I 70# simple beam if the allowable shearing unit stress is 13,000#/in^2?

59-F. A 12″ WF 25# simple beam 20′ 0″ in length supports three concentrated loads of 3,000#, 4,000#, and 5,000# at 2′ 0″, 8′ 0″, and 11′ 0″ respectively from the left-hand support. Investigate the beam for shear.

60. Deflection. It is important that a beam, in addition to resisting flexure and shear, does not deflect excessively. Floor and ceiling cracks may result if the beams are not stiff enough, and beams should be investigated to see that the deflection does not exceed $\frac{1}{360}$ of the span, the generally accepted limit. It frequently happens that a beam may be of adequate dimensions to resist bending and shear but, upon investigation, may be found to deflect more than the maximum permitted by building codes.

For typical beams and loads the actual deflection may be computed from the formulas given in Table 4, but in using these formulas note carefully that l, in the term l^3, is in inches, not feet. For a simple beam having a uniformly distributed load the deflection is found by the equation

$$D = \frac{5}{384} \times \frac{Wl^3}{EI} \quad \text{(Table 4)}$$

in which D = the maximum deflection, in inches
W = the total uniformly distributed load, in pounds
l = the length of the span, in inches
E = the modulus of elasticity, in pounds per square inch
I = the moment of inertia of the cross section of the beam, in inches to the fourth power.

For a beam whose loading is not typical we may find W, the uniformly distributed load that would produce the same bending moment, and then apply the above formula to find the deflection. (Note the example for the girder under Art. 61.) Occasionally it is convenient to compute the deflections due to individual loads on the beam, their sum being the total deflection.

A convenient rule of thumb concerning the deflection of steel beams uniformly loaded is that *the depth of the beam in inches should be at least ½ the span in feet.* For instance, a beam having a span of 20′ with a uniformly distributed load should have a depth of at

Table 22. Deflection Coefficients for Uniformly Distributed Loads

Span, Feet	f, Pounds per Square Inch		Span, Feet	f, Pounds per Square Inch		Span, Feet	f, Pounds per Square Inch	
	18,000	20,000		18,000	20,000		18,000	20,000
6	0.670	0.744	18	6.033	6.696	30	16.759	18.602
7	0.912	1.004	19	6.722	7.460	31	17.894	19.862
8	1.191	1.322	20	7.449	8.268	32	19.067	21.164
9	1.508	1.674	21	8.211	9.114	33	20.278	22.508
10	1.862	2.068	22	9.012	10.002	34	21.526	23.894
11	2.253	2.500	23	9.850	10.934	35	22.870	25.386
12	2.681	2.976	24	10.725	11.904	36	24.132	26.786
13	3.146	3.492	25	11.638	12.918	37	25.516	28.322
14	3.649	4.050	26	12.588	13.972	38	26.888	29.846
15	4.189	4.650	27	13.574	15.066	39	28.319	31.434
16	4.767	5.292	28	14.599	16.204	40	29.793	33.070
17	5.381	5.974	29	15.660	17.382	41	31.301	34.744

Coefficient for concentrated load at center of span = 0.8 of above values.
Coefficient for triangular loading, apex at center of span = 0.96 of above values.
Coefficient for equal concentrated loads at ⅓ points = 1.02 of above values.
Coefficient for irregular loading = 0.92 of above values (approximate).

Compiled from data in "Kidder-Parker Architects' and Builders' Handbook."

least $\frac{20}{2}$ or 10″. This is, of course, merely an approximation, for the actual deflection depends on several factors.

The simplest method of computing the deflection of steel beams, uniformly loaded, is to use Table 22 and the formula

$$D = \frac{\textit{coefficient in table}}{2 \times c}$$, D being the maximum deflection in inches and c the distance of the most remote fiber to the neutral surface.

To find the deflection, first find the coefficient corresponding to the span and f, the extreme fiber stress, then divide this coefficient by 2 times the distance of the most remote fiber from the neutral surface. For symmetrical sections such as I-beams and channels, c equals ½ the depth; therefore we simply divide the table coefficient by the depth of the section. Note that this table is to be used directly for uniformly distributed loads, but by multiplying the coefficients by the factors given in the footnote to the table the deflection for various types of loading may be found.

EXAMPLES

Example. A simple beam 24′ in length has a total uniformly distributed load of 18,000#. What size of beam should be used, the deflection not to exceed 1/360 of the span?

Solution. Designing the beam for flexure,

$$M = \frac{Wl}{8} = \frac{18{,}000 \times 24 \times 12}{8} = 648{,}000''\#$$

$$\frac{M}{f} = S = \frac{648{,}000}{18{,}000} = 36\text{in}^3, \text{ the required section modulus}$$

Select 12″ I 31.8# ($S = 36.0\text{in}^3$, Table 5).

Investigating for shear,

$$\text{Total load} = 18{,}000\#$$

therefore

$$R_1 = R_2 = \frac{18{,}000}{2} = 9{,}000\#$$

$$V = 9{,}000\#$$

$$A = 12 \times 0.35 = 4.2\text{in}^2 \quad \text{(Table 5)}$$

$$f_s = \frac{V}{A} = \frac{9{,}000}{4.2} = 2{,}142\#/\text{in}^2$$

Since the average shearing unit stress is less than 12,000#/in², the allowable, the beam is acceptable for shear.

Investigating for deflection,

$$D = \frac{5}{384} \times \frac{Wl^3}{EI} \quad \text{(Table 4)}$$

Data: $W = 18{,}000\#$, $l = (24 \times 12)''$, $E = 29{,}000{,}000\#/\text{in}^2$, $I = 215.8\text{in}^4$. Substituting,

$$D = \frac{5 \times 18{,}000 \times 24 \times 12 \times 24 \times 12 \times 24 \times 12}{384 \times 29{,}000{,}000 \times 215.8} = 0.88'',$$

the actual deflection

$$\text{Allowable deflection} = \frac{24 \times 12}{360} = 0.8''$$

Since the actual deflection exceeds the allowable, the beam is unacceptable. To meet the requirement for deflection, accept a 14″ WF 30#.

EXAMPLE. A simple beam having a span of 12′ and a uniformly distributed load of 15,000# is supported by an 8″ I 20.5#. Compute the deflection by use of the Table 22.

SOLUTION.

$$M = \frac{Wl}{8} = \frac{15{,}000 \times 12 \times 12}{8} = 270{,}000''\#$$

To find the actual f,

$$\frac{M}{f} = S \quad \text{or} \quad f = \frac{M}{S}$$

The section modulus of an 8″ I 20.5# is 15.1in³ (Table 5). Then

$$f = \frac{270{,}000}{15.1} = 17{,}880\#/\text{in}^2$$

The coefficient in Table 22 for a span of 12′ and a unit stress f of 18,000#/in² is 2.681.

To be accurate, the coefficient for $f = 17{,}880$ is $\frac{17{,}880}{18{,}000} \times 2.681 = 2.663$. $D = \frac{coefficient}{2 \times c} = \frac{2.663}{8} = 0.333''$, the actual deflection. Since the actual fiber stress is so nearly 18,000 the deflection could be said to be $\frac{2.681}{8} = 0.335''$, which is sufficiently accurate.

PROBLEMS

60-A. Letting $f = 18{,}000\#/in^2$, design a steel beam for a span of 18′ 0″ having a uniformly distributed load of 14,500#, the deflection to be within the allowable limit.

60-B. A steel beam 15′ in length has two concentrated loads of 14,000# each at the third points. Letting $f = 18{,}000\#/in^2$, design the beam and compute its deflection. (See footnote to Table 22.)

60-C. Letting $f = 18{,}000\#/in^2$, design a beam 16′ in length having a concentrated load of 10,000# at the center of the span. Compute the deflection.

60-D. Compute the deflection of a 16″ WF 36#, having a span of 18′ 0″ and a concentrated load of 14,000# at the center of the span.

60-E. Using an allowable extreme fiber stress of $20{,}000\#/in^2$, design a steel beam 20′ 0″ long supporting three concentrated loads of 5,000#, 6,000#, and 7,000# at 6′ 0″, 9′ 0″, and 14′ 0″ respectively from the left support, the deflection to be within the allowable limit.

60-F. Compute the deflection of a 15″ I 42.9# simple beam having two concentrated loads of 12,000# each at the third points of a 21′ 0″ span.

60-G. Using an allowable extreme fiber stress of $20{,}000\#/in^2$, design a steel beam 20′ 0″ in length having a total uniformly distributed load of 29,000#. Test for horizontal shear and deflection.

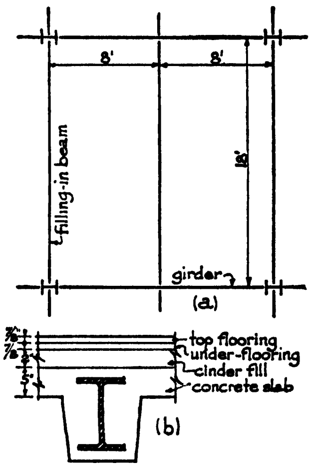


Fig. 50

61. Floor Framing. Steel framing for floor construction consists of "filling-in" beams which frame into girders, the girders framing into columns. Generally there are one, two, or three filling-in beams per bay. In most types of floor construction the floor load is carried directly by the beams, and the loads

on the girders are those which are transferred from the beams, the beams having uniformly distributed loads and the girders concentrated loads. A typical example is given below.

EXAMPLE

EXAMPLE. Design the steel framing for the floor bay shown in Fig. 50, the live load to be 100#/ft^2 and the floor construction as shown in the section.

SOLUTION.

Floor loads		
Live load	=	100#/ft^2
Top floor	=	3
Under floor	=	3
2″ cinder fill	=	16
5″ concrete slab	=	60
Movable partitions	=	18
Total	=	200#/ft^2 of floor

Beams are 8′ apart; therefore $200 \times 8 = 1{,}600\#$, the floor load on each linear foot of beam. Approximate weight of the beam and its fireproofing = 120 pounds per linear foot; $1{,}600 + 120 = 1{,}720\#$, the total load per linear foot acting on one beam.

$$1{,}720 \times 18 = 30{,}960\#, \text{ the total load on one beam}$$

Designing the beam for bending,

$$M = \frac{Wl}{8} = \frac{30{,}960 \times 18 \times 12}{8} = 835{,}920''\#,$$

the maximum bending moment

$$\frac{M}{f} = S = \frac{835{,}920}{18{,}000} = 46.4\text{in}^3, \text{ the required section modulus}$$

Select a 12″ WF 40#.

Testing for shear,

$$R_1 = R_2 = \frac{30{,}960}{2} = 15{,}480\# = V, \text{ the maximum shear}$$

Area of web of beam $= 11.94 \times 0.294 = 3.51\text{in}^2$ (see Table 6)

$$f_s = \frac{V}{A} = \frac{15{,}480}{3.51} = 4{,}410\#/\text{in}^2, \text{ the average shearing unit stress}$$

This being less than 12,000#/in², the allowable, the beam is acceptable for shear.

Testing for deflection. According to the rule given in Art. 59, the depth of the beam in inches should be at least ½ the span in feet, or the depth should be at least $\frac{18}{2} = 9''$. However, the beam we have selected has 12″, a greater depth, so that the deflection is probably within the allowable limit.

$$\text{Allowable deflection} = \frac{18 \times 12}{360} = 0.6''$$

The actual deflection is computed by the formula $D = \frac{5}{384} \times \frac{Wl^3}{EI}$ (Table 4).

$$D = \frac{5 \times 30{,}960 \times 18 \times 12 \times 18 \times 12 \times 18 \times 12}{384 \times 30{,}000{,}000 \times 310.1} = 0.41'',$$

the actual deflection

Therefore the beam is acceptable.

DESIGNING THE GIRDER

Loads. The total load on one of the filling-in beams was found to be 30,960#, the reactions at each end of a beam being $\frac{30{,}960}{2} =$ 15,480#. The girder has a span of 16′ and a concentrated load at the center. Note carefully that this concentrated load is the sum of the reactions from the beams framing into the girder on each side, or 2 × 15,480 = 30,960#. The only uniformly distributed load will be the weight of the girder and its fireproofing. Assume this to be 140 pounds per linear foot.

$$\text{Total load on girder} = 30{,}960 + (16 \times 140) = 33{,}200\#$$

$$R_1 = R_2 = \frac{33{,}200}{2} = 16{,}600\# = V, \text{ the maximum shear}$$

Designing the beam for bending,

$M_{(x=8)} = (16{,}600 \times 8) - (140 \times 8 \times 4) = 128{,}320'\# = 1{,}539{,}840''\#$

$$\frac{M}{f} = S = \frac{1{,}539{,}840}{18{,}000} = 85.5\text{in}^3, \text{ the required section modulus}$$

Select a 14″ WF 61#.

Testing for shear. The area of the web of the section selected $= 13.91 \times 0.378 = 5.25\text{in}^2$. $f_s = \dfrac{V}{A} = \dfrac{16{,}600}{5.25} = 3{,}161\#/\text{in}^2$, the average shearing unit stress. The girder is acceptable for shear since the average shearing stress is less than 12,000#/in².

Testing for deflection. There is no formula in Table 4 for this loading; therefore compute W, the uniformly distributed load which would produce the same bending moment.

$$M = \frac{Wl}{8}$$

$$128{,}320 = \frac{W \times 16}{8}$$

$$W = 64{,}160\#$$

$$D = \frac{5}{384} \times \frac{Wl^3}{EI}$$

$$D = \frac{5 \times 64{,}160 \times 16 \times 12 \times 16 \times 12 \times 16 \times 12}{384 \times 30{,}000{,}000 \times 641.5} = 0.3''$$

the actual deflection. The allowable deflection $= \dfrac{16 \times 12}{360} =$ 0.53″; therefore the girder is satisfactory.

PROBLEMS

61-A. Using an allowable extreme fiber stress of 18,000#/in², design the beams and girders for the floor framing shown in Fig. 51 (*a*) allowing a live load of 60#/ft². The floor construction is shown in Fig. 51 (*b*). Allow 38#/ft² for the weight of a 10″ tile arch.

61-B. A floor panel in a steel frame building has columns spaced 20′ 0″ o.c. both ways. The beams are spaced 5′ 0″ o.c. and they frame into the girders

at the quarter points. The floor construction consists of 3″ plank flooring and a ⅞″ finished flooring. The live load is 50#/ft². Design the beams, using an allowable extreme fiber stress of 20,000#/in².

61-C. Design the girders for the floor panel described in Prob. 61-B, using the same data.

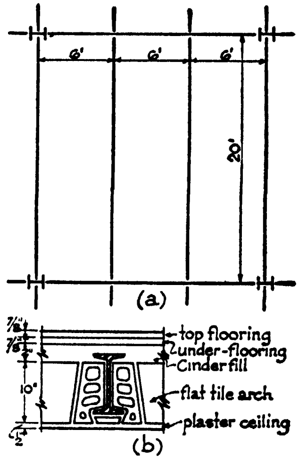


FIG. 51

62. **Designing Steel Beams.** In designing beams the loads are never known accurately because, when the computations are begun, the size of the beam, and consequently its weight, are unknown. One method employed is to approximate the load due to the beam itself and to check this assumed load with the actual weight of the beam selected as a result of computations. Many designers, in designing relatively small beams, ignore the weight due to the beam and select a beam whose section modulus is slightly larger than that found by using the flexure formula. Practice in such computations soon acquaints one with the increase to be made in the section modulus due to the weight of the beam.

From the examples previously given it is seen that the actual shearing unit stress has been considerably less than the allowable. This is commonly the case. For the usual uniformly loaded beam it is seldom necessary to test for shear. However, beams of relatively short spans with large loads, or beams having concentrated loads close to a support, should be tested. Often the shearing stress is the determining factor in designing grillage beams. In practice, beams having uniformly distributed loads are selected directly from safe load tables such as Tables 23-A and 23-B.

63. **Beams Laterally Supported.** If a beam is not supported laterally by the floor construction, or by other beams framing into it, it is possible to have failure due to buckling of the upper flange. If the extreme fiber stress in bending is not to exceed 18,000#/in² or 20,000#/in², the length of the beam should not exceed 15 times the width of the flange unless lateral support is provided. In most

Table 23-A. Safe Uniformly Distributed Loads in Kips for Various Beams and Spans *

Based on an extreme fiber stress, f = 18,000#/in²

Size and Type	Weight per Foot, Lb	Section Modulus, In³	Span in Feet—Beams Fixed Laterally																		
			6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
6" I	12.5	7.3	14.5	12.4	10.9	9.7	8.7	7.9	7.3	6.7											
6" I	14.75	7.9	15.9	13.6	11.9	10.6	9.5	8.7	7.9	7.3											
6" I	17.25	8.7	17.3	14.9	13.0	11.6	10.4	9.5	8.7	8.0											
7" I	15.3	10.4	20.7	17.7	15.5	13.8	12.4	11.3	10.4	9.6	8.9	8.3									
7" I	17.5	11.1	22.3	19.1	16.7	14.8	13.4	12.1	11.1	10.3	9.5	8.9									
7" I	20.0	12.0	24.0	20.5	18.0	16.0	14.4	13.1	12.0	11.1	10.3	9.6									
8" WF	17.0	14.1	28.2	24.2	21.2	18.8	16.9	15.4	14.1	13.0	12.1	11.3	10.6	10.0							
8" I	18.4	14.2	28.4	24.4	21.3	19.0	17.1	15.5	14.2	13.1	12.2	11.4	10.7	10.0							
8" I	20.5	15.1	30.1	25.8	22.6	20.1	18.1	16.4	15.1	13.9	12.9	12.0	11.3	10.6							
8" WF	19.0	16.0	32.0	27.4	24.0	21.3	19.2	17.5	16.0	14.8	13.7	12.8	12.0	11.3							
8" I	23.0	16.0	32.1	27.5	24.1	21.4	19.3	17.5	16.0	14.8	13.7	12.8	12.0	11.3							
8" I	25.5	17.0	34.0	29.2	25.5	22.7	20.4	18.6	17.0	15.7	14.6	13.6	12.8	12.0							
8" WF	21.0	18.0	36.0	30.9	27.0	24.0	21.6	19.6	18.0	16.6	15.4	14.4	13.5	12.7							
8" WF	24.0	20.8	41.6	35.7	31.2	27.7	25.0	22.7	20.8	19.2	17.8	16.6	15.6	14.7							
10" WF	21.0	21.5	43.0	36.9	32.3	28.7	25.8	23.5	21.5	19.8	18.4	17.2	16.1	15.2	14.3	13.6	12.9				
8" WF	27.0	23.4	46.8	40.1	35.1	31.2	28.1	25.5	23.4	21.6	20.1	18.7	17.6	16.5							
10" WF	23.0	24.1	48.2	41.3	36.2	32.1	28.9	26.3	24.1	22.2	20.7	19.3	18.1	17.0	16.1	15.2	14.5				
10" I	25.4	24.4	48.8	41.9	36.6	32.6	29.3	26.6	24.4	22.5	20.9	19.5	18.3	17.2	16.3	15.4	14.7				
10" I	30.0	26.7	53.4	45.8	40.1	35.6	32.0	29.1	26.7	24.6	22.9	21.4	20.0	18.8	17.8	16.9	16.0				
8" WF	31.0	27.4	54.8	47.0	41.1	36.5	32.9	29.9	27.4	25.3	23.5	21.9	20.6	19.3							
10" WF	26.0	27.6	55.2	47.3	41.4	36.8	33.1	30.1	27.6	25.5	23.7	22.1	20.7	19.5	18.4	17.4	16.6	15.8			
10" I	35.0	29.2	58.3	50.0	43.7	38.9	35.0	31.8	29.2	26.9	25.0	23.3	21.9	20.6	19.4	18.4	17.5				
8" WF	33.0	29.3	58.0	50.2	44.0	39.1	35.2	32.0	29.3	27.0	25.1	23.4	22.0	20.7							
10" WF	29.0	30.8	61.6	52.8	46.2	41.1	37.0	33.6	30.8	28.4	26.4	24.6	23.1	21.7	20.5	19.5	18.5	17.6			
12" WF	25.0	30.9	61.8	53.0	46.4	41.2	37.1	33.7	30.9	28.5	26.5	24.7	23.2	21.8	20.6	19.5	18.5	17.7	16.9	16.1	15.5
10" I	40.0	31.6	63.2	54.2	47.4	42.1	37.9	34.5	31.6	29.2	27.1	25.3	23.7	22.3	21.1	20.0	19.0				
10" WF	33.0	35.0	68.3	60.0	52.5	46.7	42.0	38.2	35.0	32.3	30.0	28.0	26.3	24.7	23.3	22.1	21.0				
12" WF	28.0	35.6	69.1	61.0	53.4	47.5	42.7	38.8	35.6	32.9	30.5	28.5	26.7	25.1	23.7	22.5	21.4	20.3	19.4	18.6	17.8
12" I	31.8	36.0	71.9	61.7	54.0	48.0	43.2	39.2	36.0	33.2	30.8	28.8	27.0	25.4	24.0	22.7	21.6	20.6	19.6	18.8	18.0
12" I	35.0	37.8	75.7	64.9	56.8	50.5	45.4	41.3	37.8	34.9	32.4	30.3	28.4	26.7	25.2	23.9	22.7	21.6	20.6	19.7	18.9
10" WF	37.0	39.9	72.6	68.4	59.9	53.2	47.9	43.5	39.9	36.8	34.2	31.9	29.9	28.2	26.6	25.2	23.9				
12" WF	32.0	40.7	79.4	69.8	61.1	54.3	48.8	44.4	40.7	37.6	34.9	32.6	30.5	28.7	27.1	25.7	24.4	23.3	22.2	21.2	20.4
14" WF	30.0	41.8	83.6	71.7	62.7	55.7	50.2	45.6	41.8	38.6	35.8	33.4	31.4	29.5	27.9	26.4	25.1	23.9	22.8	21.8	20.9
10" WF	41.0	44.5	78.7	76.3	66.8	59.3	53.4	48.5	44.5	41.1	38.1	35.6	33.4	31.4	29.7	28.1	26.7				
12" I	40.8	44.8	89.7	76.8	67.2	59.8	53.8	48.9	44.8	41.4	38.4	35.9	33.6	31.6	29.9	28.3	26.9	25.6	24.4	23.4	22.4
12" WF	36.0	45.9	89.6	78.7	68.9	61.2	55.1	50.1	45.9	42.4	39.3	36.7	34.4	32.4	30.6	29.0	27.5	26.2	25.0	23.9	23.0
12" I	45.0	47.3	94.7	81.2	71.0	63.1	56.8	51.6	47.3	43.7	40.6	37.9	35.5	33.4	31.6	29.9	28.4	27.1	25.8	24.7	23.7
14" WF	34.0	48.5	96.4	83.1	72.8	64.7	58.2	52.9	48.5	44.8	41.6	38.8	36.4	34.2	32.3	30.6	29.1	27.7	26.5	25.3	24.3

* Loads to the left of the dotted vertical lines will produce maximum allowable shear in webs.
Loads to the right of heavy vertical lines will produce excessive deflections

Table 23-A (Continued)

Size and Type	Weight per Foot, Lb	Section Modulus, In^3	Span in Feet—Beams Fixed Laterally 12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
10" WF	45.0	49.1	49.1	45.3	42.1	39.3	36.8	34.7	32.7	*31.0*	*29.5*	*28.1*									
12" I	50.0	50.3	50.3	46.4	43.1	40.2	37.7	35.5	33.5	31.8	30.2	28.7	*27.4*	*26.2*	*25.1*						
12" WF	40.0	51.9	51.9	47.9	44.5	41.5	38.9	36.6	34.6	32.8	31.1	29.7	*28.3*	*27.1*	*26.0*						
12" I	55.0	53.2	53.2	49.1	45.6	42.6	39.9	37.6	35.5	33.6	31.9	30.4	*29.0*	*27.8*	*26.6*						
14" WF	38.0	54.6	54.6	50.4	46.8	43.7	41.0	38.5	36.4	34.5	32.8	31.2	29.8	28.5	27.3	26.2	*25.2*	*24.3*	*23.4*		
16" WF	36.0	56.3	56.3	52.0	48.3	45.0	42.2	39.7	37.5	35.6	33.8	32.2	30.7	29.4	28.2	27.0	26.0	25.0	24.1	*23.3*	*22.5*
12" WF	45.0	58.2	58.2	53.7	49.9	46.6	43.7	41.1	38.8	36.8	34.9	33.3	*31.7*	*30.4*	*29.1*						
15" I	42.9	58.9	58.9	54.4	50.5	47.1	44.2	41.6	39.3	37.2	35.3	33.7	32.1	30.7	29.5	28.3	27.2	*26.2*	*25.2*	*24.4*	
15" I	45.0	60.5	60.5	55.8	51.8	48.4	45.3	42.7	40.3	38.2	36.3	34.5	33.0	31.5	30.2	29.0	27.9	*26.9*	*25.9*	*25.0*	
14" WF	42.0	60.7	60.7	56.0	52.0	48.6	45.5	42.8	40.5	38.3	36.4	34.7	33.1	31.7	30.4	29.1	*28.0*	*27.0*	*26.0*		
14" WF	43.0	62.7	62.7	57.9	53.7	50.2	47.0	44.3	41.8	39.6	37.6	35.8	34.2	32.7	31.4	*30.1*	*28.9*	*27.9*			
15" I	50.0	64.2	64.2	59.2	55.0	51.3	48.1	45.3	42.8	40.5	38.5	36.7	35.0	33.5	32.1	30.8	29.6	*28.5*	*27.5*	*26.5*	
16" WF	40.0	64.4	64.4	59.4	55.2	51.5	48.3	45.5	42.9	40.7	38.6	36.8	35.1	33.6	32.2	30.9	29.7	28.6	27.6	*26.6*	*25.8*
12" WF	50.0	64.7	64.7	59.7	55.5	51.8	48.5	45.7	43.1	40.9	38.8	37.0	*35.3*	*33.8*	*32.4*						
15" I	55.0	67.8	67.8	62.6	58.1	54.3	50.9	47.9	45.2	42.8	40.7	38.8	37.0	35.4	33.9	32.6	31.3	*30.1*	*29.1*	*28.1*	
14" WF	48.0	70.2	70.2	64.8	60.2	56.2	52.7	49.6	46.8	44.3	42.1	40.1	38.3	36.6	35.1	*33.7*	*32.4*	*31.2*			
12" WF	53.0	70.7	70.7	65.3	60.6	56.6	53.0	49.9	47.1	44.7	42.4	40.4	*38.6*	*36.9*	*35.4*						
16" WF	45.0	72.4	72.4	66.8	62.1	57.9	54.3	51.1	48.3	45.7	43.4	41.4	39.5	37.8	36.2	34.8	33.4	32.2	31.0	*30.0*	*29.0*
16" WF	50.0	80.7	80.7	74.5	69.2	64.6	60.5	57.0	53.8	51.0	48.4	46.1	44.0	42.1	40.4	38.7	37.2	35.9	34.6	33.4	*32.3*
15" I	60.8	81.2	81.2	75.0	69.6	65.0	60.9	57.3	54.1	51.3	48.7	46.4	44.3	42.4	40.6	39.0	37.5	*36.1*	*34.8*	*33.6*	
18" WF	47.0	82.3	82.3	76.0	70.5	65.8	61.7	58.1	54.9	52.0	49.4	47.0	44.9	42.9	41.2	39.5	38.0	36.6	35.3	34.1	32.9
15" I	65.0	84.3	84.3	77.8	72.2	67.4	63.2	59.5	56.2	53.2	50.6	48.2	46.0	44.0	42.1	40.5	38.9	*37.5*	*36.1*	*34.9*	
18" I	54.7	88.4	88.4	81.6	75.8	70.7	66.3	62.4	58.9	55.8	53.0	50.5	48.2	46.1	44.2	42.4	40.8	39.3	37.9	36.6	35.4
18" WF	50.0	89.0	89.0	82.2	76.3	71.2	66.8	62.8	59.3	56.2	53.4	50.9	48.5	46.4	44.5	42.7	41.1	39.6	38.1	36.8	35.6
14" WF	61.0	92.2	92.2	85.1	79.0	73.8	69.2	65.1	61.5	58.2	55.3	52.7	50.3	48.1	46.1	*44.3*	*42.6*	*41.0*			
18" I	60.0	93.1	93.1	85.9	79.8	74.5	69.8	65.7	62.1	58.8	55.9	53.2	50.8	48.6	46.5	44.7	43.0	41.4	39.9	38.5	37.2
16" WF	58.0	94.1	94.1	86.9	80.7	75.3	70.6	66.4	62.7	59.4	56.5	53.8	51.3	49.1	47.1	45.2	43.4	41.8	40.3	*38.9*	*37.6*
18" I	65.0	97.5	97.5	90.0	83.6	78.0	73.1	68.8	65.0	61.6	58.5	55.7	53.2	50.9	48.8	46.8	45.0	43.3	41.8	40.4	39.0
18" WF	55.0	98.2	98.2	90.6	84.2	78.6	73.7	69.3	65.5	62.0	58.9	56.1	53.6	51.2	49.1	47.1	45.3	43.6	42.1	40.6	39.3
18" I	70.0	101.9	101.9	94.1	87.4	81.6	76.5	72.0	68.0	64.4	61.2	58.3	55.6	53.2	51.0	48.9	47.1	45.3	43.7	42.2	40.8
14" WF	68.0	103.0	103.0	95.1	88.3	82.4	77.3	72.7	68.7	65.1	61.8	58.9	56.2	53.7	51.5	49.4	*47.5*	*45.8*	*44.1*		
16" WF	64.0	104.2	104.2	96.2	89.3	83.4	78.2	73.6	69.5	65.8	62.5	59.5	56.8	54.4	52.1	50.0	48.1	46.3	44.7	*43.1*	*41.7*
18" WF	64.0	117.0	117.0	108.0	100.3	93.6	87.8	82.6	78.0	73.9	70.2	66.9	63.8	61.0	58.5	56.2	54.0	52.0	50.1	48.4	46.8
18" WF	70.0	128.2	128.2	118.3	109.9	102.6	96.2	90.5	85.5	81.0	76.9	73.3	69.9	66.9	64.1	61.5	59.2	57.0	54.9	53.0	51.3
18" WF	77.0	141.7	141.7	130.8	121.5	113.4	106.3	100.0	94.5	89.5	85.0	81.0	77.3	73.9	70.9	68.0	65.4	63.0	60.7	58.6	56.7
16" WF	88.0	151.3	151.3	139.7	129.7	121.0	113.5	106.8	100.9	95.6	90.8	86.5	82.5	78.9	75.7	72.6	69.8	67.2	64.8	*62.6*	*60.5*
18" WF	85.0	156.1	156.1	144.1	133.8	124.9	117.1	110.2	104.1	98.6	93.7	89.2	85.1	81.4	78.1	74.9	72.0	69.4	66.9	64.6	62.4
16" WF	96.0	166.1	166.1	153.3	142.4	132.9	124.6	117.2	110.7	104.9	99.7	94.9	90.6	86.7	83.1	79.7	76.7	73.8	71.2	68.7	*66.4*

* Loads to the right of heavy vertical lines will produce excessive deflections.

Table 23-B. Safe Uniformly Distributed Loads in Kips for Various Beams and Spans
Based on an extreme fiber stress, $f = 20{,}000\#/in^2$

Size and Type	Weight per Foot, Lb	Section Modulus, In^3	Span in Feet—Beams Fixed Laterally																	
			8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
8 WF	17.0	14.1	24	21	18.8	17.1	15.7	14.5	13.4	12.5	11.7	11.1									
8 I	18.4	14.2	24	21	18.9	17.2	15.8	14.6	13.5	12.6	11.8	11.1									
8 I	20.5	15.1	25	22	20	18.3	16.8	15.5	14.4	13.4	12.6	11.8									
8 I	23.0	16.0	27	24	21	19.4	17.8	16.4	15.2	14.2	13.3	12.5									
8 I	25.5	17.0	28	25	23	21	18.9	17.4	16.2	15.1	14.2	13.3									
8 WF	21.0	18.0	30	27	24	22	20	18.5	17.1	16.0	15.0	14.1									
8 WF	24.0	20.8	35	31	28	25	23	21	19.8	18.5	17.3	16.3									
10 WF	21.0	21.5	36	32	29	26	24	22	21	19.1	17.9	16.9	15.9	15.1	14.3	13.7					
8 WF	27.0	23.4	39	35	31	28	26	24	22	21	19.5	18.4									
10 I	25.4	24.4	41	36	33	30	27	25	23	22	20	19.1	18.1	17.1	16.3	15.5					
10 I	30.0	26.7	45	40	36	32	30	27	25	24	22	21	19.8	18.7	17.8	17.0					
10 WF	26.0	27.6	46	41	37	34	31	28	26	25	23	22	20	19.4	18.4	17.5					
10 I	35.0	29.2	49	43	39	35	32	30	28	26	24	23	22	21	19.5	18.5					
12 WF	25.0	30.9	52	46	41	38	34	32	29	28	26	24	23	22	21	19.6	18.7	17.9	17.2	16.5	
10 I	40.0	31.6	53	47	42	38	35	32	30	28	26	25	23	22	21	20					
10 WF	33.0	35.0	58	52	47	42	39	36	33	31	29	28	26	25	23	22					
12 WF	28.0	35.6	59	53	48	43	40	37	34	32	30	28	26	25	24	23	22	21	19.8	19.0	
12 I	31.8	36.0	60	53	48	44	40	37	34	32	30	28	27	25	24	23	22	21	20	19.2	
12 I	35.0	37.8	63	56	50	46	42	39	36	34	32	30	28	27	25	24	23	22	21	20	
12 WF	32.0	40.7	68	60	54	49	45	42	39	36	34	32	30	29	27	26	25	24	23	22	
14 WF	30.0	41.8	70	62	56	51	46	43	40	37	35	33	31	29	28	27	25	24	23	22	21
10 WF	41.0	44.5	74	66	59	54	49	46	42	40	37	35	33	31	30	28					
12 I	40.8	44.8	75	66	60	54	50	46	43	40	37	35	33	31	30	28	27	26	25	24	
12 WF	36.0	45.9	77	68	61	56	51	47	44	41	38	36	34	32	31	29	28	27	26	25	
12 I	45.0	47.3	79	70	63	57	53	49	45	42	39	37	35	33	32	30	29	27	26	25	
14 WF	34.0	48.5	81	72	65	59	54	50	46	43	40	38	36	34	32	31	29	28	27	26	25
12 I	50.0	50.3	84	75	67	61	56	52	48	45	42	40	37	35	34	32	31	29	28	27	
12 WF	40.0	51.9	87	77	69	63	58	53	49	46	43	41	38	36	35	33	32	30	29	28	
12 I	55.0	53.2	89	79	71	65	59	55	51	47	44	42	39	37	36	34	32	31	30	28	
14 WF	38.0	54.6	91	81	73	66	61	56	52	49	46	43	40	38	36	35	33	32	30	29	28
16 WF	36.0	56.3	94	83	75	68	63	58	54	50	47	44	42	40	38	36	34	33	31	30	29
12 WF	45.0	58.2	97	86	78	71	65	60	55	52	49	46	43	41	39	37	35	34	32	31	
15 I	42.9	58.9	98	87	79	71	65	60	56	52	49	46	44	41	39	37	36	34	33	31	30
15 I	45.0	60.5	101	90	81	73	67	62	58	54	50	48	45	43	40	38	37	35	34	32	31
14 WF	43.0	62.7	105	93	84	76	70	64	60	56	52	49	46	44	42	40	38	36	35	33	32
15 I	50.0	64.2	107	95	86	78	71	66	61	57	54	50	48	45	43	41	39	37	36	34	33
16 WF	40.0	64 4	107	95	86	78	72	66	61	57	54	51	48	45	43	41	39	37	36	34	33
15 I	55.0	67.8	113	100	90	82	75	70	65	60	57	53	50	48	45	43	41	39	38	36	35

Table 23·B (Continued)

Size and Type	Weight per Foot, Lb	Section Modulus, In^3	Span in Feet—Beams Fixed Laterally																		
			15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
14 WF	48.0	70.2	62	59	55	52	49	47	45	43	41	39	37	36	35	33	32	31			
12 WF	53.0	70.7	63	59	56	52	50	47	45	43	41	39	38								
16 WF	45.0	72.4	64	60	57	54	51	48	46	44	42	40	39	37	36	35	33	32	31	30	29
14 WF	53.0	77.8	69	65	61	58	55	52	49	47	45	43	42	40	38	37	36	35			
15 I	60.8	81.2	72	68	64	60	57	54	52	49	47	45	43	42	40	39	37	36	35	54	
15 I	65.0	84.3	75	70	66	62	59	56	54	51	49	47	45	43	42	40	39	38	36	35	
18 I	54.7	88.4	79	74	69	66	62	59	56	54	51	49	47	45	44	42	41	39	38	37	36
18 WF	50.0	89 0	79	74	70	66	63	59	57	54	52	49	48	46	44	42	41	40	38	37	36
14 WF	61.0	92.2	82	77	72	68	65	62	59	56	53	51	49	47	46	44	42	41			
18 I	60.0	93.¹	83	78	73	69	65	62	59	56	54	52	50	48	46	44	43	41	40	39	38
16 WF	58.0	94.1	84	78	74	70	66	63	60	57	55	52	50	48	47	45	43	42	41	39	38
18 I	65.0	97.5	87	81	77	72	68	65	62	59	57	54	52	50	48	46	45	43	42	41	39
18 WF	55.0	98.2	87	82	77	73	69	66	62	60	57	55	52	50	49	47	45	44	42	41	40
18 I	70.0	101 9	91	85	80	76	72	68	65	62	59	57	54	52	50	49	47	45	44	43	41
14 WF	68.0	103 0	92	86	81	76	72	69	65	62	60	57	55	53	51	49	47	46			
16 WF	64.0	104 2	93	87	82	77	73	70	66	63	60	58	56	53	52	50	48	46	45	43	42
18 WF	64.0	117 0	104	98	92	87	82	78	74	71	68	65	62	60	58	56	54	52	50	49	47
21 WF	59.0	119 3	106	99	94	88	84	80	76	72	69	66	64	61	59	57	55	53	51	50	48
21 WF	63.0	128 0	114	107	100	95	90	85	81	78	74	71	68	66	63	61	59	57	55	53	52
18 WF	70.0	128 2	114	107	101	95	90	86	81	78	74	71	68	66	63	61	59	57	55	53	52
16 WF	88.0	151.3	135	126	119	112	106	101	96	92	88	84	81	78	75	72	70	67	65	63	61
16 WF	96.0	166.1	148	138	130	123	117	111	106	101	96	92	89	85	82	79	76	74	71	69	67
21 WF	82.0	168 0	149	140	132	124	118	112	107	102	97	93	90	86	83	80	77	75	72	70	68
24 WF	74 0	170.4	152	142	134	126	120	114	108	103	99	95	91	87	84	81	78	76	73	71	69
18 WF	96.0	184 4	164	154	145	137	129	123	117	112	107	103	98	95	91	88	85	82	79	77	75
24 WF	80 0	185 8	165	155	146	138	130	124	118	113	108	103	99	95	92	89	85	83	80	77	75
21 WF	96.0	197.6	176	165	155	146	139	132	126	120	115	110	105	101	98	94	91	88	85	82	80
18 WF	105.0	202.2	180	169	159	150	142	135	128	123	117	112	108	104	100	96	93	90	87	84	82
27 WF	91.0	233.2	207	194	183	173	164	156	148	141	135	130	124	120	115	111	107	104	100	97	94
24 WF	100.0	248.9	221	207	195	184	175	166	158	151	144	138	133	128	123	119	114	111	107	104	101
21 WF	112.0	249.6	222	208	196	185	175	166	159	151	145	139	133	128	123	119	115	111	107	104	101
27 WF	98.0	255.3	227	213	200	189	179	170	162	155	148	142	136	131	126	122	117	114	110	106	103
21 WF	132.0	294.8	262	246	231	218	207	197	187	179	171	164	157	151	146	140	136	131	127	123	119
24 WF	120.0	299.1	266	249	235	222	210	199	190	181	173	166	160	153	148	142	138	133	129	125	121
30 WF	108.0	299.2	266	249	235	222	210	200	190	181	173	166	160	153	148	143	138	133	129	125	121
30 WF	116.0	327.9	292	273	257	243	230	219	208	199	190	182	175	168	162	156	151	146	141	137	132
24 WF	130.0	330.7	294	276	259	245	232	221	210	200	192	184	176	170	163	158	152	147	142	138	134
24 WF	150.0	385.5	343	321	302	286	271	257	245	234	224	214	206	198	190	184	177	171	166	161	156

Loads to the right of heavy vertical lines will produce excessive deflections

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types of building construction, floor slabs or other construction lend lateral support to the upper flange, though occasionally stiffener beams must be introduced.

64. **Safe Load Tables.** Since beams with uniformly distributed loads occur so frequently in practice, tables giving the maximum load for specific spans are presented for the convenience of the designer. Tables 23-A and 23-B have been compiled from tables published by the steel companies. *Note particularly that 18,000#/in² is the allowable extreme fiber stress in Table 23-A and that 20,000#/in² has been used in Table 23-B.* The loads are given in kips, a kip being 1,000#. The letters I and WF identify respectively American Standard and Wide Flange beam sections. With a few exceptions, the Bethelehem and Carnegie Steel companies roll beams of identical depths and weights. The load and span being known, the proper size section is selected directly by referring to the tables. Remember that, if the depth of the beam is not a governing factor, the lightest-weight beam is generally the most economical.

EXAMPLE

EXAMPLE. Select by use of Table 23-A a beam having a span of 14′ and a uniformly distributed load of 20,000#.

SOLUTION. By referring to the table, we find the lightest-weight beam to be a 10″ WF 23#. If we are limited in the depth of the beam, an 8″ WF 27#, is adequate, though the weight is greater. A stiffer beam, that is one having less deflection, would be a 12″ WF 25#. Note that the latter beam is stronger, having a greater section modulus, but is only 2# heavier than the 10″ WF 23#.

PROBLEMS

64-A-B-C-D-E. By means of the safe load tables select the most economical beams to support 25,000# uniformly distributed for spans of 8′, 10′, 13′, 15′, and 18′, using an allowable extreme fiber stress of 1,800#/in².

64-F-G-H-I-J. By means of the safe load tables select the most economical beams to support a uniformly distributed load of 31,000# for spans of 9′, 12′, 14′, 16′, and 20′, using an allowable extreme fiber stress of 20,000#/in².

65. **Bearing Plates for Beams.** When a beam rests upon a masonry wall it is usually necessary to provide a steel bearing plate to distribute the load over a larger area. The use of wide-flange

beams frequently results in the omission of such plates. The area of the plate is determined by dividing the load transmitted by the beam, by the allowable compressive stress of the masonry. Table 24 gives allowable bearing strengths for various kinds of masonry.

Table 24. Allowable Bearing Strength of Masonry Walls

Stone concrete, 2,000#/in^2 ultimate	500#/in^2
Common brick, lime mortar . . .	100#/in^2
Common brick, lime-cement mortar. . . .	200#/in^2
Common brick, cement mortar..	250#/in^2
Hard brick, cement mortar..	300#/in^2
Rubble, cement mortar	150#/in^2
Rubble, lime-cement mortar	100#/in^2
Hollow T. C. blocks, cement mortar.....	80#/in^2
Hollow cinder blocks, cement mortar ..	80#/in^2

Figure 52 represents a beam resting on a bearing plate having an area of $B \times C$ and a thickness of t.

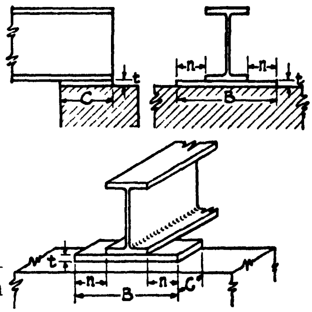


FIG. 52

R = the reaction of the beam, in pounds

B = the length of the plate, in inches

C = the width of the plate, in inches

Area of plate

$$= B \times C = \frac{R}{\text{Allowable bearing strength of the masonry}}$$

w = the actual bearing stress on the masonry, in pounds per square inch

$$n = \frac{B - (\text{width of flange})}{2} = \text{the projection of the plate on each}$$

side of the beam, in inches

f = the allowable bending stress of steel, 18,000#/in^2 or 20,000#/in^2

t = the thickness of the bearing plate, in inches.

To compute t, substitute values in the formula

$$t = \sqrt{\frac{3 \times w \times B \times n}{2 \times f}}$$

EXAMPLE

Example. A 15″ I 42.9#, having an end reaction of 31,000#, is to be supported by a wall laid up with common brick in cement mortar. If the width of the plate C is limited to 8″, design the bearing plate.

Solution. The allowable bearing strength of the brickwork is 250#/in² (Table 24). Then the area of the plate will be $\frac{31,000}{250} =$ 124in² $= B \times C$. Since C is limited to 8″, $B = \frac{124}{8} = 15.5''$. Use an 8″ x 16″ plate.

$$\text{Actual bearing on masonry } w = \frac{31,000}{8 \times 16} = 242\#/\text{in}^2$$

The width of flange of a 15″ I 42.9# is 5.5″ (Table 5). Then $n = \frac{16 - 5.5}{2} = 5.25''$. (See Fig. 52.)

$$t = \sqrt{\frac{3 \times w \times B \times n}{2 \times f}} = \sqrt{\frac{3 \times 242 \times 16 \times 5.25}{2 \times 18,000}} = \sqrt{1.69} = 1.3''$$

Use a 16″ x 8″ x 1⅜″ plate.

PROBLEMS

65-A. The total uniformly distributed load on a 10″ WF 21# is 36,400#. What should be the size of a bearing plate if C is 13″, the supporting wall is of common brick laid up in lime mortar, and $f = 18,000\#/\text{in}^2$?

65-B. Design a bearing plate for a 14″ WF 38#, having a reaction of 45,000#. The beam is supported by a wall of common brick laid in cement mortar, the width of the plate is limited to 12″, and $f = 18,000\#/\text{in}^2$.

65-C. A concrete wall of 1:2:4 mix supports an 18″ I 54.7#, having a reaction of 52,000#. Design the bearing plate, limiting its width to 8″, using 18,000#/in² for f.

65-D. A masonry wall laid up with common brick and lime-cement mortar supports one end of a 12″ I 31.8# on which there is a total uniformly distributed

load of 57,000#. Compute the size of the bearing plate if its width is limited to 12″ and the allowable extreme fiber stress is 20,000#/in^2.

65-E. A 16″ WF 40#, 15′ 0″ in length and loaded with three concentrated loads of 15,000# each at 2′ 0″, 6′ 0″, and 9′ 0″ respectively from the left end, is supported by hard-brick walls laid up in cement mortar. If the bearing plates are limited to an 8″ width, design the larger plate, using an allowable extreme fiber stress of 20,000#/in^2.

65-F. A 15″ I 42.9# is used for a simple beam on which there is a uniformly distributed load of 70,000#. The beam is supported by walls of common brick laid up with cement mortar. Design the beam bearing plates, which are limited to a width of 9″, using an allowable extreme fiber stress of 20,000#/in^2.

66. **Riveted Connections.** Two plates held together by one or more rivets, as shown in Fig. 53 (*a*), is called a lap joint; it is the simplest type of riveted connection. A more common type in building construction consists of three thicknesses of metal, Fig.

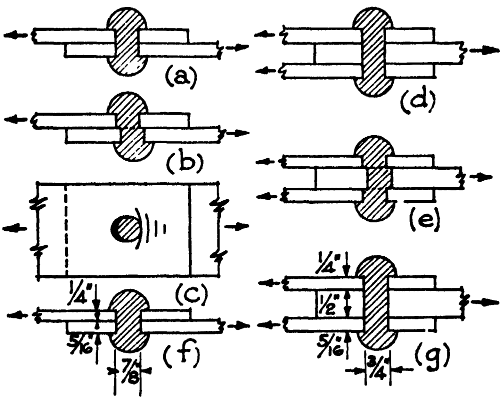


FIG. 53

53 (*d*). In order to determine the number of rivets required for a connection we must first determine the allowable working value for a single rivet. Then the load at the connection divided by the allowable working value of one rivet gives the number of rivets required.

There are several different ways in which a riveted joint may fail but we shall be concerned with only the two major causes, shear and bearing. The allowable working value for shear for the rivet in Fig. 53 (*b*) is the cross-section area of the rivet multiplied by the allowable shearing unit stress of the rivet steel. This rivet is in *single shear*. If there are three thicknesses of metal, as in

Fig. 53 (*e*), the rivet is in *double shear* and therefore, since there are two planes of the rivet to fail by shearing, the allowable working value for shear is 2 times the cross-section area of the rivet multiplied by the allowable shearing unit stress of the rivet steel.

If the plates are relatively thin with respect to the diameter of the rivet, failure may occur by crushing of the plate by the bearing pressure exerted by the rivet. The area assumed to resist bearing is a rectangle the dimensions of which are the diameter of the rivet and the thickness of the plate. Therefore the allowable bearing value of a rivet is this area multiplied by the allowable bearing unit stress. When rivets are in single shear as in Fig. 53 (*a*) the rivets are also in *single bearing.*

Table 25-A. Allowable Working Values for Power-Driven Rivets

Unit shearing stress = 13,500#/in²

Unit bearing stress { single bearing = 24,000#/in²; double bearing = 30,000#/in² }

Diameter of rivet	$\frac{5}{8}$ in		$\frac{3}{4}$ in		$\frac{7}{8}$ in		1 in		$1\frac{1}{8}$ in	
Area of rivet	0.3068		0 4418		0.6013		0 7854		0 9940	
Single shear pounds	4,140		5,960		8,120		10,600		13,420	
Double shear pounds	8,280		11,930		16,240		21,200		26,840	
Bearing	Single Lb	Double Lb	Single Lb	Double Lb	Single Lb	Double Lb	Single Lb	Double Lb	Single Lb	Double Lb
Thickness of plate in inches: $\frac{1}{4}$	3,750	4,690	4,500	5,630	5,250	6,560	6,000	7,500	6,750	8,440
$\frac{5}{16}$	....	5,860	5,630	7,030	6,560	8,200	7,500	9,380	8,440	10,550
$\frac{3}{8}$	....	7,030	....	8,440	7,880	9,840	9,000	11,250	10,130	12,660
$\frac{7}{16}$	....	8,200	....	9 840	....	11,480	10,500	13,130	11,810	14,770
$\frac{1}{2}$	....	.....	....	11,250	....	13,130	.. ..	15,000	.....	16,880
$\frac{9}{16}$	....	.....	....	.....	....	14,770	.....	16,880	.....	18,980
$\frac{5}{8}$	....	.....	....	.....	....	.....	.....	18,750	.....	21,090

In Fig. 53 (*d*) the rivet in the two outer plates is in single bearing, but the middle or enclosed plate is said to be in *double bearing.*

Owing to the more nearly uniform distribution of stresses and partly to the friction between the plates, the bearing value of a rivet in a plate tightly enclosed between two others is greater than when there are but two plates, as in Fig. 53 (*a*).

Table 25-B. Allowable Working Values for Power-Driven Rivets

Unit shearing stress = 15,000#/in²

Unit bearing stress { single bearing = 32,000#/in²; double bearing = 40,000#/in² }

Diameter of Rivet		5/8″		3/4″		7/8″		1″		1 1/8″	
Area of Rivet		0.3068		0.4418		0.6013		0.7854		0.9940	
Single Shear, Pounds		4,600		6,630		9,020		11,780		14,910	
Double Shear, Pounds		9,200		13,250		18,040		23,560		29,820	
Bearing		Single, Lb	Double, Lb	Single, Lb	Double, Lb	Single, Lb	Double, Lb	Single, Lb	Double, Lb	Single, Lb	Double, Lb
Thickness of plate in inches	1/4	....	6,250	6,000	7,500	7,000	8,750	8,000	10,000	9,000	11,250
	5/16	....	7,810	....	9,380	8,750	10,900	10,000	12,500	11,300	14,100
	3/8	....	9,380	....	11,300	....	13,100	12,000	15,000	13,500	16,900
	7/16	....	.....	....	13,100	....	15,300	.....	17,500	.....	19,700
	1/2	....	. .		..	. ..	17,500	....	20,000	.....	22,500
	9/16	....	.		.		.	....	22,500	.....	25,300
	5/8	....	.. ..	.. .	.....	....	.....	.....	.....	.....	28,100

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Many building codes permit the following stresses for power-driven rivets.

Unit shearing stress	13,500#/in²
Single-bearing unit stress	24,000#/in²
Double-bearing unit stress	30,000#/in²

Table 25-A gives the allowable working stresses for rivets of various diameters in plates of different thicknesses. The values have been computed in accordance with the above unit stresses,

and the use of this table greatly simplifies the design of a riveted connection. Note that bearing values are not given beyond the point where shear governs. For direct stresses the examples below, using the above stresses, illustrate the method of computing allowable working values.

The current specifications of the American Institute of Steel Construction permit the following stresses for power-driven rivets.

Unit shearing stress	15,000#/in^2
Single-bearing unit stress	32,000#/in^2
Double-bearing unit stress	40,000#/in^2

Table 25-B has been compiled in accordance with these stresses.

EXAMPLES

Example. Compute the allowable working stress for a 7/8" rivet connecting 1/4" and 5/16" plates. See Fig. 53 (*f*).

Solution. The area of a 7/8" rivet is πr^2 or $3.1416 \times (7/16)^2 = 0.6013\text{in}^2$. The rivet being in single shear, the allowable shearing value is $0.6013 \times 13,500 = 8,120\#$.

The area resisting bearing in the 1/4" plate is $(1/4 \times 7/8)\text{in}^2$; therefore the allowable bearing value $= (1/4 \times 7/8) \times 24,000 = 5,250\#$. For the 5/16" plate, the area $= (5/16 \times 7/8)\text{in}^2$; consequently the allowable bearing value $= (5/16 \times 7/8) \times 24,000 = 6,560\#$. Of these three allowable working values, 8,120#, 5,250#, and 6,560#, 5,250# is the smallest. This is called the controlling value. If, for instance, we have 1/4" and 5/16" plates riveted with 7/8" rivets and the load transferred from one plate to the other is 27,000#, we divide the load by the controlling value of one rivet to find the required number of rivets. Then $\frac{27,000}{5,250} = 5+$. Therefore we use 6 rivets.

These figures show the method of computing the allowable working values, but *the actual computations are unnecessary as the working values may be found directly by referring to Table 25-A.* See whether the values given in the table agree with the above computations.

Example. A ¾″ rivet connects three plates whose thicknesses are ¼″, ⁵⁄₁₆″, and ½″ as shown in Fig. 53 (*g*). Compute the allowable working values.

Solutions. The rivet is in double shear, and since its cross-section area is 0.4418in^2, the allowable shearing value for double shear is 0.4418 × 13,500 × 2 = 11,930#.

The ¼″ plate is in single bearing, the allowable bearing value being ¼ × ¾ × 24,000 = 4,500#.

Similarly, the ⁵⁄₁₆″ plate has an allowable bearing value of ⁵⁄₁₆ × ¾ × 24,000 = 5,630#. In this example the ½″ plate is enclosed between two others, that is, the rivet here is in double bearing, and the allowable bearing value for double bearing is 30,000#/in^2. Therefore the allowable bearing value is ½ × ¾ × 30,000 = 11,250#. For this particular connection the controlling value is 4,500#. Compare these values with those given in Table 25-A.

Examples of computing the number of rivets in joints of roof trusses are given in Art. 114.

PROBLEMS

66-A. Two ½″ plates are held together with ⅞″ rivets as in Fig. 53 (*a*). Compute the controlling working value of the rivet, and check the result with the values in Table 25-A.

66-B. What is the controlling value of the rivet in the previous problem if the rivet is ¾″ in diameter?

66-C. A ⅞″ rivet connects three plates as in Fig. 53 (*d*). The outer plates are ⅜″, and the enclosed plate ½″. Compute the controlling value of the rivet, and check the values given in Table 25-A.

66-D. A ⁵⁄₁₆″ plate and a ¼″ plate are connected by ¾″ rivets. If the load transferred from one plate to the other is 15,550# and the unit stresses given in Table 25-B are used, how many rivets are required?

66-E. Two ¼″ plates are connected by ⅞″ rivets. The load transferred from one plate to the other is 26,250#. What number of rivets is needed if the stresses used in Table 25-B are employed?

66-F. Rivets 1″ in diameter are used to connect three plates as shown in Fig. 53 (*e*). The outer plates are ⁵⁄₁₆″, and the inner plate ⁷⁄₁₆″ in thickness. If the load transferred by the plates is 62,000# and the stresses in Table 25-B are used, how many rivets are needed?

67. **Beam Connections.** Figure 54 illustrates typical framed connections for floor beams. The most common type of connection consists in riveting two connecting angles to the web of the beam,

the outstanding legs of the angles then being riveted to the girder or column. The seat angle shown in Fig. 54 (*a*) is not counted on to support the beam but is merely used as an aid in erection. In order to bring a spandrel beam nearer to the outer face of the wall the connection shown in Figs. 54 (*e*) and 54 (*f*) is often employed.

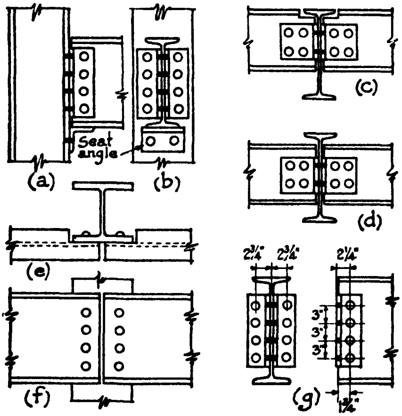


FIG. 54

It is unnecessary for the designer to compute the proper size of connecting angles and the number of rivets for the usual framed connection. *Standard beam connections* have been adopted for all sizes of beams and may be found by referring to any steel manufacturer's handbook. As an example, two 4″ x 3½″ x ⅜″ angles 11½″ in length with ⅞″ rivets, as shown in Fig. 54 (*g*), is the standard connection for certain 15″ and 18″ I-beams.

68. **Plate Girders.** Whenever possible, rolled beams, for economy, are used in building construction. It frequently happens, however, that rolled beams of adequate dimensions are not available for long spans with relatively large loads. This necessitates the use of built-up sections called plate or box girders. A plate girder consists of a deep plate, called the *web*, at the top and bottom of which are riveted *flange angles*. One or more plates, depending on the area required, are riveted to the outstanding legs of the flange angles and are known as *cover plates*. In general a plate girder takes the form of an I-beam section. In addition to the flange angles, *stiffener angles*, placed in a vertical position, are

riveted to the sides of the web. These angles reinforce the web against buckling and are placed under concentrated loads and at points of support. Stiffener angles placed at other points in the length of the girder are called *intermediate stiffeners.* The *depth* of a plate girder is generally the distance back to back of the flange angles, and the *effective depth* is the vertical distance between centers of gravity of the upper and lower flanges.

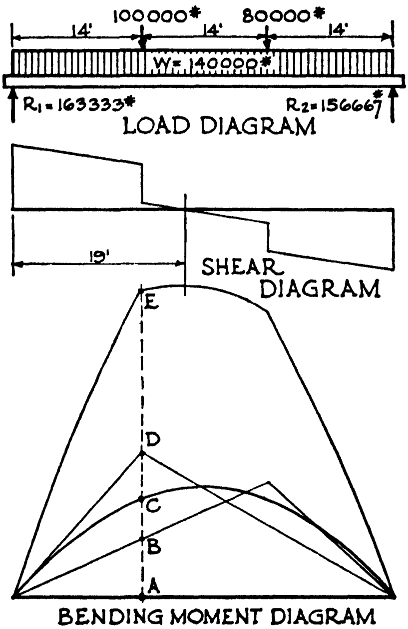


Fig. 55

In designing plate girders two fundamental principles are accepted, namely:

First, the function of the web is to resist the shearing stresses. For convenience in computations it is assumed that the shearing stresses are distributed uniformly over the cross-section area of the web.

Second, the function of the flanges, which consist of the flange angles and cover plates, is to resist compression and tension stresses due to bending. Some designers compute ⅛ of the web area as flange area in each flange. In a book of this scope it is

inadvisable to enter into discussions of the many details and modifications of plate-girder design, and although many different methods are employed, the following illustrative problem explains very briefly methods which may be used. The subsequent articles explain the various steps in the solution of the example stated herewith.

EXAMPLE

EXAMPLE. Design a plate girder 42′ long having a uniformly distributed load of 140,000# over its entire length and in addition two concentrated loads of 100,000# and 80,000# each, placed at the third points of the span. See Fig. 55.

69. Shear and Moment Diagrams. The first step in the design of a plate girder is to compute the reactions and make diagrams showing the values of the shear and bending moments. See Arts. 23, 28, 30, and 31.

SOLUTION. To compute the reactions,

$$42R_1 = (140{,}000 \times 21) + (100{,}000 \times 28) + (80{,}000 \times 14)$$

$$R_1 = 163{,}333\#$$

$$R_2 = (140{,}000 + 100{,}000 + 80{,}000) - 163{,}333$$

$$R_2 = 156{,}667\#$$

The shear diagram is drawn as has been explained in Art. 28. See Fig. 55.

The moment diagram may be drawn as explained in Arts. 30 and 31. Another method, which may be shorter, follows. Compute separately the three reactions for R_1 and R_2 due to the three loads, 140,000#, 100,000#, and 80,000#. See Fig. 55. The maximum moment for the uniformly distributed load alone is at the center of the span. Its value is

$$M = \frac{Wl}{8} = \frac{140{,}000 \times 42}{8} = 735{,}000'\# = 8{,}820{,}000''\#$$

A convenient method of drawing a parabola for the moment diagram for the uniformly distributed load is shown in Fig. 56 (a). Draw AB, the girder length. Since the value of M is 8,820,000″# and occurs at the center of the span, plot the point C at any con-

venient scale. The problem now is to draw a parabola connecting the points A, C, and B. The graphical method of determining various points in the curve is readily seen by inspecting the diagram.

The maximum moment for each concentrated load taken separately occurs directly under the load, and this value is computed for each of the two loads, the respective reactions previously found

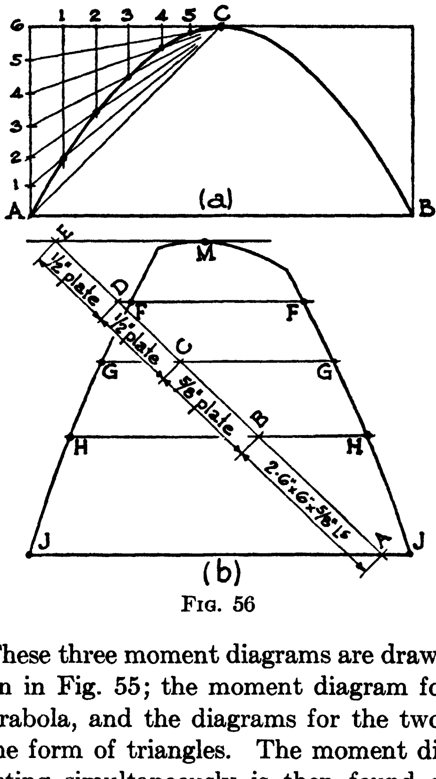


FIG. 56

being used. These three moment diagrams are drawn one over the other as shown in Fig. 55; the moment diagram for the uniform load is the parabola, and the diagrams for the two concentrated loads are in the form of triangles. The moment diagram for the three loads acting simultaneously is then found graphically by adding the values of the individual moments at various points in the length of the beam. For instance, the value of the bending moment under the 100,000# load is AE, found by adding the vertical distances AB, AC, and AD.

From the shear diagram we find that the value of zero shear is at some point between the two concentrated loads. To find this distance from the left reaction we write

$$0 = 163{,}333 - \left(100{,}000 + \frac{140{,}000}{42}x\right)$$

$$163{,}333 = 100{,}000 + 3{,}333x$$

$$x = 19' \quad \text{(See second example in Art. 31.)}$$

This determines the position of the maximum bending moment The value of the maximum bending moment is

$$M_{(x=19)} = (163{,}333 \times 19) - \left[\left(\frac{140{,}000}{42} \times 19 \times \frac{19}{2}\right) + (100{,}000 \times 5)\right]$$

$$= 2{,}001{,}901'\# = 24{,}022{,}812''\#$$

The value of the maximum vertical shear is the greater reaction or $V = 163{,}333\#$.

70. Web Plate. The depth of the plate girder must be assumed by the designer. In building construction existing conditions frequently determine the depth, but in general $\frac{1}{8}$ to $\frac{1}{14}$ the span length proves to be the most economical depth, $\frac{1}{12}$ being common practice.

Shallow beams provide greater lateral stiffness, but beams of greater depth have less deflection. It is customary to make the distance from back to back of flange angles $\frac{1}{2}''$ greater than the depth of the web plate.

The minimum thickness of the web is generally accepted to be $\frac{3}{8}''$. Since the function of the web is to resist shear, the value of the maximum shear may determine this thickness. A requirement frequently found in specifications is that the thickness shall not be less than $\frac{1}{170}$ of the unsupported distance between flanges. When the unsupported distance between flanges is greater than 60 times the web thickness, stiffeners shall be required at distances not greater than 6′ apart.

Solution. Assume the depth to be $\frac{1}{12}$ the span length. Then $D = \frac{42 \times 12}{12} = 42''$. Since the stock width of plates is usually in even inches, let us assume the plate to be 42″ deep and, adding $\frac{1}{2}''$ to provide for any irregularities in the edge, we determine the distance back to back of the flange angles, $42\frac{1}{2}''$.

The web plate is to resist shear; therefore

$$A = \frac{V}{v}$$

in which A = the cross-section area of the web plate, in square inches

V = the maximum vertical shear, in pounds

v = the allowable shearing unit stress, in pounds per square inch

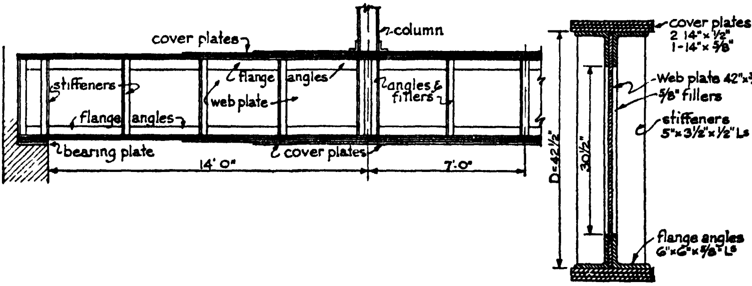


FIG. 57

Substituting, $A = \frac{163{,}333}{12{,}000}$, A = 13.61in^2, the area of the plate. The depth of the plate being 42″, the thickness required to resist shear will be $\frac{13.61}{42}$ = 0.32″. This is slightly less than ⅜″, the minimum thickness permitted; therefore we shall accept ⅜″.

Here is another check for thickness. Assume the flange angles to be 6″ x 6″ x ⅝″. This gives 42½ − (6 + 6) = 30½″, the unsupported distance between flanges. See Fig. 57. The plate thickness should not be less than 1/170 of this distance, or $\frac{30.5}{170}$ = 0.179″. The ⅜″ plate is greater than this requirement; therefore we shall accept a ⅜″ plate 42″ in depth.

71. **Flanges. Cover Plates.** The simplest method in the design of flanges for plate girders is the application of the plate-girder formula

$$M = fAd \qquad \text{or} \qquad A = \frac{M}{fd}$$

in which M = the maximum bending moment, in inch-pounds
f = the allowable bending fiber stress, in pounds per square inch
A = the required cross-section area of the flange, in square inches
d = the effective depth, the distance between centers of gravity of the flanges, in inches.

The flanges are usually composed of two angles and one or more cover plates. In proportioning the area of the angles and cover plates no infallible rule can be made but it is good practice to require that the center of gravity of the flange be kept within the back of the angles. It is also well to have no cover plates thicker than the angles.

SOLUTION. In the formula $A = \frac{M}{fd}$, d is the effective depth or distance between centers of gravity of the flanges. This distance must be assumed to begin with. It is usually taken about 1″ less than the depth of the web plate. In this problem d will be $42 - 1 = 41''$. Substituting in the formula $A = \frac{M}{fd}$,

$$A = \frac{24{,}022{,}800}{18{,}000 \times 41} = 32.5\text{in}^2$$

Assuming the angles to be 6″ x 6″ x ⅝″, each angle has an area of 7.11in^2 (Table 10), and the area of two angles is $2 \times 7.11 = 14.22\text{in}^2$. The area to be made up by cover plates therefore is

$$32.5 - 14.22 = 18.28\text{in}^2$$

If the width of the cover plates is assumed to be 14″, the total thickness will be $18.28 \div 14 = 1.305''$. This can be made up of three 7⁄16″ plates, $3 \times 0.4375 = 1.3125''$.

These plates would be adequate for the upper flange but, since the lower flange is in tension, extra material must be added to take the place of the material lost in the rivet holes. A ¾″ rivet

requires a ⅞″ diameter hole; therefore the material lost by one hole passing through three 7/16″ plates and one ⅝″ angle will be

$$(1.3125 + 0.625) \times 0.875 = 1.695\text{in}^2$$

If failure should occur as the result of tension in the lower flange, the line of fracture would include two holes; hence the area lost by rivet holes is $2 \times 1.695 = 3.39\text{in}^2$. This must be made up in the thickness of the plates. As noted above, 18.28in^2 was to be made up by cover plates; therefore $18.28 + 3.39 = 21.67\text{in}^2$, the area required by cover plates for the lower flange, and $\frac{21.67}{14} =$ 1.54″, the minimum thickness required. This thickness can be supplied by one ⅝″ plate and two ½″ plates, $0.625 + 1 = 1.625''$. It is customary to use the same thickness of plates for both upper and lower flanges. The most recent specification of the American Institute of Steel Construction permits that no deduction shall be made for rivet holes in either flange except for special cases where the areas of the flange rivet holes in either flange exceeds 15 per cent of the gross flange area. If 15 per cent is exceeded, the excess is deducted in computing the effective area.

The bending moment used in the formula $M = fAd$ was the maximum, occurring at a distance of 19′ from the left reaction. The flanges just computed provide material to resist bending stresses at this point, but it is uneconomical to continue all the cover plates to the ends of the girder since the bending moment diminishes in magnitude on each side of the critical section. The required length of the plates may be readily found by graphical methods.

Figure 56 (*b*) shows the bending-moment diagram for the combined loads. Draw a horizontal line through M, the point of maximum moment, and from a point on the base line draw at any convenient scale the line AE equal to 36.97 units in length, representing the area of flange angles and cover plates. Divide the line AE into four parts AB, BC, CD, and DE representing respectively the areas of the two 6″ x 6″ x ⅝″ angles, one 14″ x ⅝″ plate, and two 14″ x ½″ plates. These areas are 14.22, 8.75, 7, and 7, their sum being 36.97in^2. Through the points D, C, and B draw horizontal lines intersecting the moment curve. Then F–F gives the

theoretical length of the top cover plate, *G–G* the length of the next ½″ cover plate, and *H–H* the length of the ⅝″ plate. The two uppermost cover plates are extended on each side 1′ 6″ or 2′ beyond the theoretical points to provide for riveting, and it is customary to extend the plate in direct contact with the angles the full length of the girder. The flange angles also extend to points *J–J*, the length of the girder.

72. Stiffeners. To prevent buckling of the web, stiffeners are used. They consist of angles having one leg riveted to the web plate and the outstanding leg fitting tightly between the upper and lower flange angles. Stiffener angles are used in pairs, one on each side of the web. There are two distinct classes of stiffeners, those occurring at supports or under concentrated loads, and those occurring between such loads, called intermediate stiffeners.

Solution. The stiffener angles at the support resist the maximum reaction, 163,333#. Assuming an allowable compressive stress of 18,000#/in², $\frac{163,333}{18,000} = 9.07\text{in}^2$, the required area of the outstanding legs of the angles. Four 5″ x 3½″ x ½″ angles provide 10in² in the outstanding legs and are acceptable for this problem. The greater of the two concentrated loads is 100,000#.

$$\frac{100,000}{18,000} = 5.5\text{in}^2$$

Use four 4″ x 3½″ x ⅜″ angles.

Some codes require that intermediate stiffeners be used when the unsupported distance between flanges is greater than 60 times the thickness of the web; $60 \times \frac{3}{8} = 22.5''$. Since this is less than 30.5″, the unsupported distance for this particular example, intermediate stiffeners will be required; they will be 4″ x 3½″ x ⅜″ angles. The spacing of such stiffeners should not exceed the full depth of the girder.

73. Rivets in Stiffeners. The number of rivets in stiffeners is found by application of the formula

$$n = \frac{P}{C}$$

in which n = the number of rivets
P = the concentrated load or reaction, in pounds
C = the controlling value of one rivet, in pounds.

The assumption is that each rivet takes an equal share of the load.

SOLUTION. Considering the stiffeners at the supports, the greater reaction is 163,333#. The web is 3/8″, and the stiffener angles are 5″ x 3½″ x ½″. The rivets will be ¾″ diameter. Since each rivet passes through two angle legs, the web plate and two fillers (see Fig. 57), the controlling value will be that of double shear (see Table 25-A). Then $n = \frac{P}{C}$ or $n = \frac{163{,}333}{11{,}930} = 14.$ Since there are two pairs of stiffener angles at the support, each stiffener will have a minimum of 7 rivets.

74. **Rivets in Flange Angles.** The flange angles have a tendency to slide along the web plate, and the rivets connecting these members resist this action. This stress is actually horizontal shear and of course varies in intensity throughout the length of the plate girder. The method of procedure is to compute the pitch of the rivets for the sections between stiffeners or panels. This spacing is found by the formula

$$p = \frac{C \times d_c}{V}$$

in which p = the pitch of the rivets, in inches
C = the controlling value of a rivet, in pounds
d_c = the distance between centers of gravity of the upper and lower flange angles, in inches
V = the vertical shear at a particular section of the girder, in pounds.

SOLUTION. The first stiffener occurs at 3′ from the left reaction. The value of V at this section,

$$V = 163{,}333 - \left(\frac{140{,}000}{42} \times 3\right) = 153{,}333\#$$

The distance between centers of gravity of the flange angles is

$$d_c = 42.5 - (2 \times 1.73) = 39.04''$$

The controlling value of a ¾″ rivet connecting two 6″ x 6″ x ⅝″ angles and the ⅜″ web plate is 8,440#.

$$p = \frac{C \times d_c}{V} = \frac{8{,}440 \times 39.04}{153{,}333} = 2.2''$$

Use 2¼″ pitch. The pitch of the rivets for the adjacent panels will be greater since the value of the shear decreases up to the point of maximum bending moment.

75. **Steel Columns.** Building codes permit an allowable compressive stress of either 18,000 or 20,000#/in^2 for structural steel. A small block 1″ x 1″ x 1″ will support such loads safely, but it is obvious that a bar 1in^2 in cross section having a length of, say, 30″ will support a smaller load safely because of the tendency to bend; see Fig. 58. Therefore in columns the element of slenderness must be taken into consideration. In timber columns the term *slenderness ratio* is $\frac{l}{d}$, the length in inches divided by the dimension of the least side. In steel columns $\frac{l}{r}$ represents the slenderness ratio, l, as before, being the length of the column in inches, but r being the least radius of gyration. A short column or block fails by crushing, but long slender columns fail generally by stresses due to bending. In short members the average unit stress $f = \frac{P}{A}$, but in columns long enough to have a tendency to bend the stresses are not equally distributed over the cross section and therefore the *average* unit stress must be less than 18,000#/in^2. This average stress is dependent on the slenderness ratio, $\frac{l}{r}$.

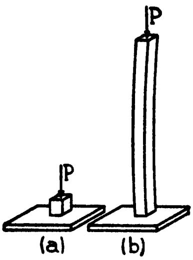


FIG. 58

76. **Column Sections.** Owing to the tendency to bend, the safe load on a column depends not only on the number of square inches in the cross section but also on the manner in which the material is distributed with respect to the axes of the cross section. That is, the *shape* of the column section is an important element. A column tends to bend in a plane perpendicular to the axis of the

cross section about which the moment of inertia is the least. Since column cross sections are seldom symmetrical with respect to both major axes, the ideal section would be one in which the moment of inertia for each major axis is equal. Pipe columns meet this condition, but their use is limited because of difficulties in making beam connections.

Of the two major axes of an I-beam, the moment of inertia about the axis parallel to the web is much the smaller; hence, for the amount of material in the cross section, I-beams are not economical sections when used as columns or struts. In former years built-up sections such as Figs. 59 (*b*) and 59 (*c*) were used extensively, but WF sections, Fig. 59 (*a*), are now rolled in numerous sizes and are used universally. They are frequently called H-columns. For excessive loads or unusual conditions, plates are riveted to the flanges of WF columns to give added strength. In steel trusses, members having compressive stresses are invariably formed of two angles as shown in Fig. 59 (*d*).

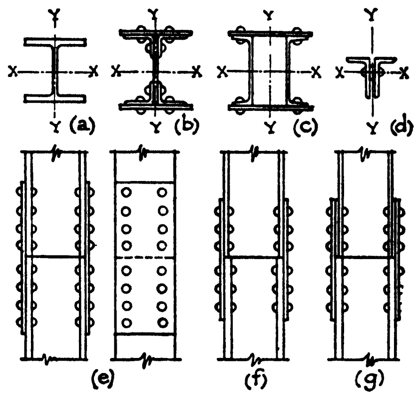


FIG. 59

In steel frame buildings, columns generally extend two stories in height. Splice lines, that is, joints between column sections, are usually about 2′ above the floor line to permit space for beam and girder connections. See Figs. 59 (*c*), 59 (*f*), and 59 (*g*).

77. Slenderness Ratio. In the design of steel columns it is important to remember that the *least* radius of gyration r is the one to be considered. It is dependent on the area and shape of

the cross section and is a measure of effectiveness in resisting bending. As seen in Art. 44, $r = \sqrt{\frac{I}{A}}$; hence, if we know the least moment of inertia and area of the cross section, r may be readily computed. Actually this computation is seldom necessary since this element is found in the tables of properties of steel sections. Note that in Tables 10 and 11, the *least* radius of gyration is found with respect to the diagonal axis Z–Z.

The slenderness ratio $\frac{l}{r}$ for main compression members should not exceed 120. For "secondary members," such as members receiving only stresses due to wind loads, the slenderness ratio should not exceed 200, the exact requirements depending on the prevailing building code or specification. In selecting a column one of the first steps is to compute its slenderness ratio for use in the column formula.

78. **Column Formulas.** One of the common column formulas in use today is that in which the average permissible stress is

$$f = \frac{18{,}000}{1 + \frac{1}{18{,}000}\frac{l^2}{r^2}}$$

with a maximum stress of 15,000#/in^2. In using this formula the limiting value of $\frac{l}{r}$ is 120 for main compression members and 200 for bracing and other secondary members.

The most recent American Institute of Steel Construction specification for steel columns permits an allowable unit stress on the gross column section of

$$f = 17{,}000 - 0.485\frac{l^2}{r^2}$$

for axially loaded columns with values of $\frac{l}{r}$ not greater than 120.

For axially loaded columns with values of $\frac{l}{r}$ greater than 120, the allowable unit stress on the gross column section is

$$f = \frac{18{,}000}{1 + \frac{1}{18{,}000}\frac{l^2}{r^2}}$$

These formulas will be referred to as the A.I.S.C. column formulas and will be used in the following examples.

In all the above column formulas

f = the allowable average stress, in pounds per square inch
l = the unsupported length of the column, in inches
r = the least radius of gyration with respect to the axis about which the column can bend, in inches

EXAMPLES

Example. Compute the allowable axial load on an 8″ WF 35# column having an unsupported length of 14′ 0″.

Solution. Referring to Table 6 we find the following properties:

$$r_{X-X} = 3.50'' \qquad r_{Y-Y} = 2.03'' \qquad A = 10.30\text{in}^2$$

Then, since the least radius of gyration is 2.03″,

$$\frac{l}{r} = \frac{14 \times 12}{2.03} = 82.7, \text{ the slenderness ratio}$$

The allowable average stress, by use of the A.I.S.C. formula, is

$$f = 17{,}000 - 0.485\frac{l^2}{r^2} = 17{,}000 - 0.485\frac{(14 \times 12)^2}{2.03^2} = 13{,}680\#/\text{in}^2$$

Since there are 10.30in² in the cross-section area, the allowable load on the column is $P = 10.30 \times 13{,}680 = 140{,}900\#$.

Example. Compute by means of the A.I.S.C. formula for columns the allowable axial load on a built-up section 16′ in length composed of one 12″ x ½″ plate and four 5″ x 3½″ x ½″ angles having short legs against the web plate. Entire depth of section is ½″ greater than depth of plate. See Fig. 60.

Solution. Referring to Table 11, find the following data.

Area of one angle = 4in², I for X–X axis = 10in⁴, centroid is 1 66″ from back of short leg.

$$\begin{aligned}&\text{Area of plate} &&= 12 \times 0.5 = 6\\ &\text{Area of 4 angles} &&= 4 \times 4 \quad = \underline{16}\\ &\text{Total area of section} &&= 22\text{in}^2\end{aligned}$$

First compute I for the section about the $Y-Y$ axis.

$$I_{Y-Y} \text{ for plate} = \frac{bd^3}{12} = \frac{12 \times 0.5^3}{12} = 0.125\text{in}^4$$

To compute I_{Y-Y} for the angles we use the equation for transferring moments of inertia from one axis to another. See Art. 42.

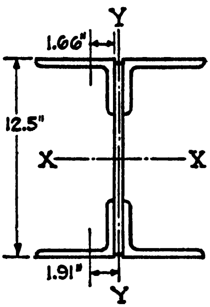


FIG. 60

Note that the centroid of the angle to the back of the short leg is 1.66″; therefore the distance of the centroid to $Y-Y$ axis of the entire cross section is $1.66 + 0.25 = 1.91''$.

$$I_{Y-Y} \text{ for 1 angle} = I + Ah^2$$

$$= 10 + (4 \times 1.91^2) = 24.58\text{in}^4$$

$$I_{Y-Y} \text{ for 4 angles} = 4 \times 24.58 = 98.32\text{in}^4$$

Therefore I_{Y-Y} for the entire area $= 98.32 + 0.125 = 98.44\text{in}^4$. By the same method, I_{X-X} of the entire cross section $= 544\text{in}^4$. Since I_{Y-Y} is smaller than I_{X-X}, the former will give the least radius of gyration, which of course is the one used in computing the allowable load on the column. Then

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{98.44}{22}} = 2.11''$$

$$\text{Slenderness ratio} = \frac{l}{r} = \frac{16 \times 12}{2.11} = 91$$

The average permissible stress,

$$f = 17{,}000 - 0.485\frac{l^2}{r^2}$$

$$f = 17{,}000 - 0.485\frac{(16 \times 12)^2}{2.11^2} = 12{,}984\#/\text{in}^2$$

Allowable load on column, $P = fA = 12{,}984 \times 22 = 285{,}600\#$.

PROBLEMS

78-A. Compute, using the A.I.S.C. formula, the allowable load on a 10″ WF 77# column having a length of 20′ 0″.

78-B. Compute the allowable axial load on an 8″ WF 31# column having an unsupported length of 18′ 0″.

78-C. Using the A.I.S.C. formula, compute the safe load on a 6″ WF 20# column having an unsupported length of 12′.

78-D. By use of the A.I.S.C. formula compute the allowable load on a built-up section composed of two 12″ 25# [s and two 14″ x ½″ plates. The channels are placed 6″ back to back, and the unsupported length of the column is 32′.

78-E. Using the A.I.S.C. formula, compute the allowable axial load on a 14″ WF 87# column having a length of 13′ 0″.

78-F. Compute the allowable axial load on an 8″ WF 31# column, having an unsupported length of 17′ 0″, by use of the A.I.S.C. column formula.

78-G. Compute, by use of the A.I.S.C. formula, the allowable axial load on a 12″ WF 79# column. The unsupported length of the column is 20′ 0″.

79. **Design of Steel Columns.** The design of columns is accomplished by the trial method. Data include the load and the length of the column; the designer selects a trial cross section and, by means of one of the column formulas, computes the allowable load that it will support. If this allowable load is less than the actual load the column will be required to support, the column section assumed is too small and another section is tested in a similar manner.

Actually, the designer selects the proper size of column section directly from tables. Table 26 gives the allowable concentric loads in kips (units of 1,000#) for various sections. It has been compiled in accordance with the A.I.S.C. formula as given in Art. 78.

Tables 27, 28, and 29 give allowable axial loads on columns that consist of two angles ⅜″ apart. These three tables are based on the formula

$$f = \frac{18{,}000}{1 + \frac{1}{18{,}000}\frac{l^2}{r^2}}$$

for $\frac{l}{r}$ values both above and below 120. In reality the usual end connections make axial loading on angle sections impossible; consequently these three tables are not considered theoretically cor-

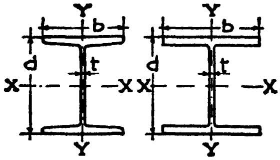
Table 26. Allowable Concentric Loads

Nominal Size	Weight, Lb	d	b	t	Area, In^2	r, In X-X	r, In Y-Y	Unsupported 6	7	8	9	10	11	12
4 x 4	10	4.00	4 000	.220	2.93	1 68	0.97	42	39	36	32	29	26	24
	13	4.16	4 060	.280	3.82	1.72	0.99	55	52	48	43	38	35	32
5 x 5	16	5 00	5 000	.240	4 70	2.13	1.26	72	70	67	63	59	55	50
	18.5	5.12	5.025	.265	5.45	2.16	1.28	84	81	78	74	69	65	59
6 x 6	15.5	6 00	6.000	.240	4 62	2 56	1.45	73	71	69	66	63	60	56
	20	6.20	6.018	.258	5 90	2.66	1.50	94	91	89	85	82	78	74
	22.5	6 28	6.050	.290	6 63	2.67	1.51	105	103	100	96	92	88	83
	25	6 37	6 080	.320	7.37	2.69	1.52	117	114	111	107	103	98	93
	27 5	6 46	6.112	.352	8.11	2.71	1.53	129	126	122	118	114	109	103
8 x 8	31	8 00	8.000	.288	9.12	3.47	2 01	149	147	145	142	139	136	132
	35	8 12	8 027	.315	10 30	3.50	2.03	169	167	164	161	158	154	150
	40	8 25	8 077	.365	11.76	3 53	2 04	193	190	187	184	180	176	172
	48	8 50	8 117	.405	14 11	3.61	2 08	232	229	225	221	217	212	207
	58	8 75	8 222	.510	17 06	3.65	2 10	280	277	273	268	263	257	251
	67	9.00	8 287	.575	19.70	3 71	2.12	324	320	315	310	304	298	291
10 x 10	49	10 00	10.000	.340	14 40	4.35	2 54	239	237	235	232	229	226	222
	54	10.12	10.028	.368	15 88	4.39	2 56	264	262	259	256	253	250	246
	66	10 38	10.117	.457	19.41	4 44	2 58	323	320	317	313	309	305	301
	77	10 62	10 195	.535	22 67	4 49	2 60	377	374	370	366	362	357	352
	89	10.88	10 275	.615	26 19	4.55	2.63	436	432	428	424	419	413	407
	100	11.12	10 345	.685	29.43	4.61	2.65	490	486	482	477	471	465	458
	112	11.38	10 415	.755	32.92	4.67	2 67	548	544	539	534	528	521	513
12 x 12	65	12.12	12.000	.390	19 11	5.28	3.02	320	318	316	313	310	307	304
	72	12.25	12 040	.430	21.16	5.31	3.04	354	352	350	347	344	341	337
	79	12 38	12.080	470	23.22	5.34	3.05	388	386	384	381	378	374	370
	92	12 62	12 155	.545	27 06	5.40	3 08	453	450	447	444	440	436	431
	106	12.88	12 230	.620	31.19	5 46	3 11	522	519	516	512	508	503	498
	133	13 38	12.365	.755	39.11	5.59	3.16	655	651	647	643	638	632	626
	161	13.88	12 515	.905	47 38	5.70	3.20	794	790	785	779	773	766	759
	190	14.38	12 670	1.060	55.86	5 82	3.25	936	932	926	920	913	905	896
14 x 14½	87	14 00	14.500	.420	25 56	6.15	3.70	430	428	426	424	422	419	416
	95	14.12	14.545	.465	27.94	6.17	3.71	470	468	466	464	461	458	455
	103	14 25	14 575	.495	30.26	6.21	3.72	509	507	505	502	499	496	493
	119	14.50	14.650	.570	34.99	6.26	3.75	589	587	584	581	578	574	570
	136	14 75	14 740	.660	39.98	6.31	3.77	673	670	667	664	660	656	651

Loads to the right of heavy vertical lines are for $\frac{l}{r}$ ratios greater than 120.

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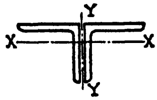
on Columns in Kips (Units of 1,000 Pounds)



Length in Feet																Bending Factor	
13	14	15	16	17	18	19	20	22	24	26	28	30	32	34	36	B_x	B_y
22	20	18	17													.704	2.139
29	26	24	22													.701	2.065
46	43	40	37	34	32	30	28									.551	1.567
54	50	47	44	41	38	36	33									.548	1.540
53	48	45	42	40	37	35	33	29	26							457	1.444
69	64	59	56	52	49	46	44	39	35							.440	1.341
78	73	67	63	59	56	53	50	44	40							.439	1.326
88	82	75	70	66	63	59	56	50	44							.439	1.316
97	90	83	78	73	69	65	62	55	49							.438	1.308
128	124	120	115	110	104	98	92	84	77	70	64	59	54			.333	.991
146	141	136	131	125	119	112	105	96	88	80	74	67	62			.331	.972
167	161	155	149	143	136	129	121	110	100	92	84	78	71	66		.331	.972
201	195	189	182	174	166	158	149	134	123	113	104	95	88	81		.327	.941
244	237	229	221	212	202	192	182	164	150	138	127	117	107	99		.328	.937
283	275	266	256	246	236	224	212	190	175	161	148	136	126	116		.326	.921
218	214	210	205	200	194	188	182	169	155	141	131	122	114	107	99	.264	.774
242	237	232	227	221	215	209	202	188	172	157	146	136	127	119	111	.263	.767
296	290	284	278	271	264	257	249	231	213	193	180	168	157	146	137	.263	.761
346	340	333	326	318	310	301	292	272	250	227	212	198	184	172	161	.263	.753
401	394	386	378	369	360	350	340	317	293	267	247	231	216	202	189	.263	.744
451	443	434	425	416	406	395	383	359	332	302	280	262	245	229	214	.262	.738
505	496	487	477	466	455	443	431	403	374	341	315	295	276	258	241	.261	.728
300	296	292	288	283	278	272	266	254	241	226	210	193	181	171	161	.217	.657
333	328	324	319	314	308	302	296	282	268	252	234	216	202	190	180	.217	.653
366	361	356	350	344	338	332	325	310	294	277	258	238	222	210	198	.217	.649
426	421	415	409	402	395	388	380	364	346	326	304	281	261	247	233	.216	.641
492	486	480	473	465	457	449	440	421	400	378	354	327	304	287	271	.216	.634
619	611	603	595	586	576	566	555	532	507	480	451	419	387	365	345	.214	.620
751	742	733	723	712	701	689	676	649	619	587	552	515	475	448	424	.213	.610
887	877	866	855	843	830	816	802	771	737	700	660	617	571	536	507	.212	.600
413	409	405	401	397	392	387	382	372	360	346	332	317	301	284	265	.185	.530
451	447	443	439	434	429	424	418	406	393	379	364	347	330	311	291	.186	.529
489	485	480	475	470	465	459	453	441	427	411	395	377	358	338	317	.185	.525
566	561	556	551	545	539	532	525	511	495	477	459	438	417	394	370	.185	.521
646	641	635	629	623	616	609	601	585	567	547	526	503	478	453	425	.185	.519

Construction.

Table 27. Struts of Two Angles



Allowable Concentric Loads in Kips for Struts of Two Equal Angles—⅜″ Back to Back

Loads to right of heavy vertical lines are for secondary members *only*

⅜″ Back to Back

Size	Thickness	Two Angles		Radius of Gyration		Axis *X–X* Unsupported length in feet													
		Area	Weight	*X–X*	*Y–Y*	3	4	5	6	7	8	9	10	12	14	16	18	20
2½ x 2½	3/16	1.80	6 1	0 78	1 18	27	27	24	22	20	18	16	14	11	...	...	...	...
	¼	2 38	8 2	0 77	1 19	36	35	32	29	26	23	21	18	15	...	...	...	...
	5/16	2 94	10 0	0 76	1 20	44	43	39	35	32	28	25	22	18	...	...	...	..
	⅜	3 46	11 8	0 75	1 21	52	51	46	41	37	33	29	26	20	...	...	...	...
3 x 3	¼	2 88	9 8	0 93	1 38	43	43	42	39	36	33	30	27	22	18	..	...	...
	5/16	3 56	12 2	0 92	1 40	53	53	52	48	44	40	36	33	27	23	...	...	...
	⅜	4 22	14 4	0 91	1 41	63	63	61	56	52	47	43	39	32	26	...	...	...
	7/16	4 86	16 6	0 91	1 42	73	73	71	65	59	54	49	45	37	30	...	...	...
3½ x 3½	5/16	4 18	14 4	1 08	1 60	63	63	63	60	56	52	48	45	38	32	27	...	...
	⅜	4 96	17 0	1 07	1 61	74	74	74	71	67	62	57	53	45	38	32	...	...
	7/16	5 74	19 6	1 07	1 62	86	86	86	83	77	71	66	61	52	44	37	...	...
	½	6 50	22 2	1 06	1 63	98	98	98	93	87	80	74	68	58	49	41	..	..
	9/16	7.24	24 8	1 05	1.64	109	109	109	103	96	89	82	76	64	54	46	..	..
	⅝	7.96	27 2	1 04	1 66	119	119	119	113	105	97	90	82	69	59	50	...	..
4 x 4	5/16	4 80	16 4	1 24	1 80	72	72	72	72	69	65	61	57	49	43	37	32	..
	⅜	5 72	19 6	1 23	1 81	86	86	86	86	82	77	72	67	59	51	44	38	...
	7/16	6 62	22 6	1 23	1 82	99	99	99	99	95	89	83	78	68	59	51	44	...
	½	7.50	25 6	1 22	1 83	113	113	113	113	107	100	94	88	76	66	57	49	...
	9/16	8 36	28 6	1 21	1 85	125	125	125	125	119	111	104	97	84	73	63	54	...
	⅝	9.22	31 4	1 20	1 86	138	138	138	138	131	122	114	107	92	79	69	59	...

rect. Nevertheless, they have been commonly used in the past and are presented here as a guide in designing compression members in trusses.

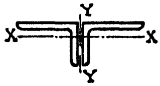
Whereas the designer may select the proper column section by merely referring to the tables, the beginner should first understand thoroughly the application of the formula by means of which the tables have been computed.

To design a steel column the design load and length are ascertained and the following steps are then taken:

STEP 1. Assume a trial section and note from the table of properties the area and r, the least radius of gyration.

Table 28. Struts of Two Angles

Allowable Concentric Loads in Kips for Struts of Two Unequal Angles—⅜″ Back to Back



Short Legs Back to Back Loads to right of heavy vertical lines are for secondary members *only*

⅜″ Back to Back

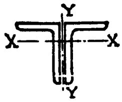
Size	Thickness	Two Angles		Radius of Gyration		Axis X–X: Unsupported length in feet										
		Area	Weight	X–X	Y–Y	3	4	5	6	7	8	9	10	12	14	16
2½ x 2	3/16	1 62	5 5	0 60	1 24	24	22	19	16	14	12	10	9	...	...	...
	¼	2 12	7 2	0 59	1 25	32	28	24	21	18	15	13	....	...	...	...
	5/16	2 62	9 0	0 58	1.26	39	34	30	25	22	19	16	....	...	...	...
3 x 2½	¼	2 62	9 0	0 75	1 45	39	38	35	31	28	25	22	20	16	...	...
	5/16	3.24	11 2	0 74	1 46	49	47	43	38	34	30	27	24	19	...	...
	⅜	3 84	13 2	0 74	1 48	58	56	51	45	40	36	32	28	22	...	...
	7/16	4 42	15 2	0 73	1 49	66	64	58	52	46	41	36	32	25	...	...
3½ x 2½	¼	2 88	9 8	0 74	1 71	43	42	38	34	30	27	24	21	17	...	...
	5/16	3 56	12 2	0 73	1 73	53	52	46	42	37	33	29	26	20	...	...
	⅜	4 22	14 4	0 72	1 74	63	61	55	49	43	38	34	30	24	...	...
	7/16	4 86	16 6	0 71	1 75	73	70	63	56	49	43	38	34	...	...	...
	½	5 50	18 8	0 70	1 76	83	78	70	62	55	48	43	38	..	...	...
4 x 3	5/16	4 18	14 4	0 89	1 93	63	63	60	55	51	46	42	37	31	25	...
	⅜	4 96	17 0	0 88	1 94	74	74	71	65	60	54	49	44	36	30	...
	7/16	5 74	19 6	0 87	1 95	86	86	82	75	68	62	56	50	41	34	...
	½	6.50	22 2	0 86	1 96	98	98	92	84	76	69	62	56	46	38	...
	9/16	7 24	24 8	0 86	1 97	109	109	103	94	85	77	69	63	51	42	...
	⅝	7 96	27 2	0 85	1.98	119	119	112	102	93	84	75	68	55	45	:
4 x 3½	5/16	4 50	15 4	1 07	1 86	68	68	68	65	60	56	52	48	40	34	29
	⅜	5 34	18 2	1 06	1 88	80	80	80	76	71	66	61	56	47	40	34
	7/16	6.18	21 2	1 05	1 88	93	93	93	88	82	76	70	64	54	46	39
	½	7.00	23 8	1 04	1 89	105	105	105	100	92	86	79	72	61	51	44
	9/16	7 80	26 6	1 03	1 90	117	117	117	110	102	95	87	80	67	57	48
	⅝	8 60	29 4	1 02	1 91	129	129	129	121	112	104	95	88	73	62	52
	11/16	9 36	32 0	1 02	1 93	140	140	140	132	122	113	104	95	80	67	57
	¾	10.12	34 6	1 01	1 94	152	152	152	142	132	121	111	102	86	72	61

Step 2. Compute the slenderness ratio $\frac{l}{r}$, l being the unsupported length of the column in inches.

Step 3. Compute f, the average allowable stress, by using a column formula as, for instance,

$$f = 17{,}000 - 0.485\frac{l^2}{r^2}$$

Table 29. Struts of Two Angles



Allowable Concentric Loads in Kips for Struts of Two Unequal Angles—3/8″ Back to Back

Long Legs Back to Back Loads to right of heavy vertical lines are for secondary members *only*

3/8″ Back to Back

Size	Thickness	Two Angles		Radius of Gyration		Axis X-X												
						Unsupported length in feet												
		Area	Weight	X-X	Y-Y	4	5	6	7	8	9	10	12	14	16	18	20	22
2½ x 2	3/16	1.62	5.5	0.79	0.92	24	22	20	18	16	14	13	10	...	...	...	...	...
	1/4	2.12	7.2	0.78	0.93	32	29	26	23	21	19	17	13	...	...	...	...	...
	5/16	2.62	9.0	0.78	0.95	39	36	32	29	26	23	20	16	...	...	...	...	...
3 x 2½	1/4	2.62	9.0	0.95	1.13	39	39	36	33	30	28	25	20	17	...	...	...	...
	5/16	3.24	11.2	0.94	1.14	49	48	44	40	37	34	31	25	21	...	...	...	...
	3/8	3.84	13.2	0.93	1.16	58	56	52	48	43	40	36	30	25	...	...	...	...
	7/16	4.42	15.2	0.92	1.17	66	64	59	54	50	45	41	34	28	...	...	...	...
3½ x 2½	1/4	2.88	9.8	1.12	1.09	43	43	42	39	37	34	32	27	23	20	17	...	...
	5/16	3.56	12.2	1.11	1.10	53	53	52	49	45	42	39	33	28	24	21	...	...
	3/8	4.22	14.4	1.10	1.11	63	63	61	57	53	50	46	39	33	28	24	...	...
	7/16	4.86	16.6	1.09	1.12	73	73	70	66	61	57	52	44	38	32	26	...	...
	1/2	5.50	18.8	1.09	1.13	83	83	80	75	69	64	59	50	43	36	31	...	...
4 x 3	5/16	4.18	14.4	1.27	1.30	63	63	63	61	57	54	50	44	38	33	29	25	...
	3/8	4.96	17.0	1.26	1.31	74	74	74	72	68	63	59	52	45	39	34	30	...
	7/16	5.74	19.6	1.25	1.32	86	86	86	83	78	73	68	60	52	45	39	34	...
	1/2	6.50	22.2	1.25	1.33	98	98	98	94	88	83	77	67	58	51	44	38	...
	9/16	7.24	24.8	1.24	1.35	109	109	109	104	98	92	86	75	65	56	49	42	...
	5/8	7.96	27.2	1.23	1.36	119	119	119	114	107	100	94	81	70	61	53	46	...
4 x 3½	5/16	4.50	15.4	1.26	1.55	68	68	68	65	61	58	54	47	41	35	31	27	...
	3/8	5.34	18.2	1.25	1.56	80	80	80	77	72	68	64	55	48	42	36	32	...
	7/16	6.18	21.2	1.24	1.57	93	93	93	89	83	78	73	64	55	48	41	36	...
	1/2	7.00	23.8	1.23	1.58	105	105	105	100	94	88	82	72	62	54	46	40	...
	9/16	7.80	26.6	1.23	1.59	117	117	117	112	105	98	92	80	69	60	52	45	...
	5/8	8.60	29.4	1.22	1.60	129	129	129	123	115	108	101	87	75	65	56	49	...
	11/16	9.36	32.0	1.21	1.62	140	140	140	133	125	117	109	94	81	70	61	53	...
	3/4	10.12	34.6	1.20	1.63	152	152	152	143	134	126	117	101	87	75	65	57	...

Step 4. Multiply *f* found in Step 3 by the number of square inches in the section. This gives the allowable load *on the column that has been assumed.*

Step 5. Compare the allowable load found in Step 4 with the design load. If the allowable load on the assumed section is less than the design load, assume a larger section and test it in the same manner.

EXAMPLES

EXAMPLE. Design a column having an unsupported length of 18′ 0″ and an axial load of 265,000#.

SOLUTION. STEP 1. Assume a 12″ WF 65# section.

From Table 6, the area = 19.11in², r_{X-X} = 5.28″ and r_{Y-Y} = 3.02″.

STEP 2. $\frac{l}{r} = \frac{18 \times 12}{3.02} = 71.5$, the slenderness ratio. Note that the *least* radius of gyration is used, and that 71.5 is within the slenderness ratio limit, 120.

STEP 3.

$$f = 17{,}000 - 0.485\frac{l^2}{r^2}$$

$$f = 17{,}000 - 0.485\frac{(18 \times 12)^2}{3.02^2} = 14{,}520\#/\text{in}^2$$

STEP 4. The allowable load P, on the column assumed, equals $f \times A$ or $P = 14{,}520 \times 19.11 = 277{,}500\#$.

STEP 5. The design load is 265,000# and, since the section assumed will safely support 277,500#, the column assumed is acceptable. Check this result with the allowable load in Table 26.

EXAMPLE. A secondary compression member in a truss is 10′ 0″ in length and has an axial load of 35,000#. Design a column section composed of two angles spaced ⅜″ apart, to provide for the gusset plate.

SOLUTION. STEP 1. Assume a section composed of two 3″ x 2½″ x ⅜″ angles, ⅜″ apart, having the long legs back to back. From Table 29, the area = 3.84in², r_{X-X} = 0.93″ and r_{Y-Y} = 1.16″.

STEP 2. $\frac{l}{r} = \frac{10 \times 12}{0.93} = 129$, the slenderness ratio.

STEP 3.

$$f = \frac{18{,}000}{1 + \frac{1}{18{,}000}\left(\frac{l}{r}\right)^2} = \frac{18{,}000}{1 + \frac{1}{18{,}000}(129)^2} = 9{,}353\#/\text{in}^2$$

STEP 4. $P = fA = 9{,}353 \times 3.84 = 36{,}000\#$, the allowable load.

STEP 5. Since the allowable load on the assumed column section is 36,000# and the design load is 35.000#, the section is acceptable.

PROBLEMS

79-A. Design, by means of the A.I.S.C. formula, a rolled section to support an axial load of 118,000# having an unsupported length of 15′ 0″. Compare the result with the loads given in Table 26.

79-B. Architectural features limit the size of a steel column to 12″ x 12″. What size of column should be used for an axial load of 308,000# if the unstayed length is 22′ 0″? Design this column by the A.I.S.C. formula and check by Table 26.

79-C. Using the A.I.S.C. formula, design a secondary compression member for a steel truss, the axial load being 37,000# and the length 12′ 0″.

79-D. Design a single angle column, 8′ 0″ in length having an axial load of 58 kips. (Refer to Table 10.)

79-E. An 8″ WF 35# column having an unsupported length of 18′ 0″ is to be used to support an axial load of 123,000#. Is this load allowable in accordance with the A.I.S.C. formula?

79-F. Using the A.I.S.C. formula, design a rolled section to support an axial load of 322,000#. The unsupported length of the column is 15′ 0″.

79-G. By use of the A.I.S.C. column formula, design a column composed of a single angle section having an unsupported length of 12′ 0″ and an axial load of 192,000#.

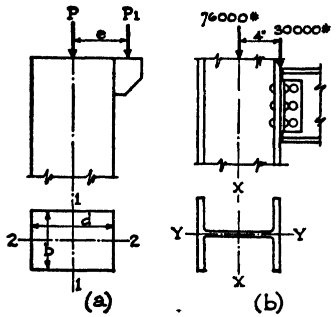


FIG. 61

79-H. A single 8″ [11.5# is used as a column to support an axial load of 34,000#. If the unsupported length is 6′ 0″, is the column safe?

80. Eccentric Loads. A load that does not coincide with the axis of a column tends to cause it to bend and is called an eccentric load, as, for instance, the beam framing into the flange of the column shown in Fig. 61 (*b*). The example at the end of this article illustrates the design procedure for such a load, but in practice engineers generally ignore this slight eccentricity in steel construction.

A convenient method employed in the design of columns having an eccentric load is to find for the eccentric load an equivalent axial load. The equivalent axial load is a load whose magnitude would produce the same maximum stress as that produced by the eccentric load.

In Fig. 61 (a), P represents an axial load and P_1 an eccentric load having an eccentricity of e. The value of an equivalent axial load for the eccentric load P_1 is $\frac{M}{k}$, in which M is the bending moment of P_1 with respect to the axis *1–1*. $M = P_1 \times e$. The term k is known as the *bending factor* or *kern distance*. Its value is found by dividing the section modulus with respect to the axis about which the eccentric load tends to bend the column by the area of the cross section, or $k = \frac{S}{A}$.*

The design of a column with an axial and eccentric load may be accomplished by use of the formula

$$f^1A = P + P_1 + \frac{M}{k}$$

in which f^1 = the maximum fiber stress in the column, not to exceed the fiber stress permitted by the column formula, in pounds per square inch

A = the area of the column cross section, in square inches

P = the axial load, in pounds

P_1 = eccentric load, in pounds

M = the bending moment of the eccentric load or $M = P_1 \times e$, in inch-pounds

k = the bending factor with respect to the axis about which the eccentric load tends to bend the column, or $k = \frac{S}{A}$, in inches.

The following steps may be used in the design of a column having an eccentric load.

STEP 1. Assume a column cross section, noting its area, section modulus, and radii of gyration.

* For a rectangular cross section as shown in Fig. 61 (a), $k = \frac{S}{A} = \frac{\frac{bd^2}{6}}{bd} = \frac{bd^2}{6} \times \frac{1}{bd} = \frac{d}{6}$. For a circular cross section, $k = \frac{d}{8}$.

STEP 2. Compute the bending moment due to the eccentric load or $M = P_1 \times e$.

STEP 3. Compute k, the bending factor of the cross section with respect to the axis of bending due to the eccentric load, $k = \frac{S}{A}$.

STEP 4. Divide M by k.

STEP 5. Compute f^1 in the equation $f^1A = P + P_1 + \frac{M}{k}$, and compare this value with the permissible value of f found by use of a column formula, such as $f = 17{,}000 - 0.485\frac{l^2}{r^2}$. $f^1 \times A$ gives the allowable load on the column section assumed.

EXAMPLE

EXAMPLE. Design a steel column 11′ 6″ in length to support an axial load of 76,000# and an eccentric load of 30,000#, Fig. 61 (*b*).

SOLUTION. STEP 1. Assume an 8″ WF 35# column. By referring to Table 6 we find $A = 10.3\text{in}^2$, $S_{X-X} = 31.1\text{in}^3$, $r_{X-X} = 3.5''$, and $r_{Y-Y} = 2.03''$.

STEP 2. $M = P_1 \times e = 30{,}000 \times 4 = 120{,}000''\#$.

STEP 3. $k = \frac{S}{A} = \frac{31.1}{10.3} = 3.01''$.

STEP 4. $\frac{M}{k} = \frac{120{,}000}{3.01} = 39{,}867$.

STEP 5.

$$f^1A = P + P_1 + \frac{M}{k} = 76{,}000 + 30{,}000 + 39{,}867 = 145{,}867\#$$

$$f^1 = \frac{145{,}867}{10.3} = 14{,}161\#/\text{in}^2$$

$$\frac{l}{r} = \frac{138}{2.03} = 68$$

Allowable stress in column formula =

$$f = 17{,}000 - 0.485\frac{l^2}{r^2}$$

$$f = 17{,}000 - 0.485\frac{138^2}{2.03^2} = 14{,}760\#/\text{in}^2$$

Since 14,161#/in^2, the stress f^1, is within the maximum stress permitted in accordance with the column formula, the assumed column section is acceptable. Note that the *least* radius of gyration (that is, about the axis Y–Y) is used in the column formula although the eccentric load tends to bend the column about the axis X–X. This is a conservative method and it errs on the side of safety. Reference to tables of safe loads for columns shows that the equivalent axial load 145,867 is acceptable for the column section assumed.

PROBLEM

80-A. Would an 8″ WF 31# be acceptable with the same data given in the above example?

81. **Reduction in Column Loads.** In a building containing a number of floors, the loads on the columns increase for each successive lower floor. These loads are made up of dead floor loads, live loads, and the weight of the columns for the floors above. Allowable live loads for various classes of buildings are given in Table 15. It is improbable, however, that the full live load will occur on all floors simultaneously. With the exception of buildings used for storage, public garages, and similar types of occupancy, building codes permit a reduction in the live loads on columns, walls, and foundations. All beams are designed for full live and dead loads.

An excerpt from the recommendations of the Building Code Committee of the Bureau of Standards, U. S. Department of Commerce follows.

Except in buildings for storage purposes, the following reductions in assumed total live floor loads are permissible in designing all columns, piers, walls, foundations, trusses, and girders:

Carrying one floor	0 per cent
Carrying two floors	10 per cent
Carrying three floors	20 per cent
Carrying four floors	30 per cent
Carrying five floors	40 per cent
Carrying six floors	45 per cent
Carrying seven or more floors	50 per cent

It is customary for the designer to tabulate in some convenient manner the loads on columns and footings. The following example is given to show one convenient method.

EXAMPLE

EXAMPLE. Tabulate the column loads on an interior column of a steel frame building 10 stories in height. The bays are 20′ x 22′, each column supporting a floor area of 440ft^2. The live floor load is 60#/ft^2, and the dead floor load, including beams and girders, 70#/ft^2. The roof has a live load of 30#/ft^2 and a dead load of 50#/ft^2. Determine the loads by the reduction of live loads recommended by the Bureau of Standards.

SOLUTION.

	Floor and Roof Loads		Design Loads
Roof—	$LL = 440 \times 30$	= 13,200	
	$DL = 440 \times 50$	= 22,000	
		35,200#	35,200#, 10th-story column
10th floor—	$LL = 440 \times 60$	= 26,400	
	$DL = 440 \times 70$	= 30,800	
	Column	= 280	
		57,480#	92,680#, 9th-story column
9th floor—	$LL = 440 \times 60 \times 0.90$	= 23,760	
	$DL = 440 \times 70$	= 30,800	
	Column	= 350	
		54,910#	147,590#, 8th-story column
8th floor—	$LL = 440 \times 60 \times 0.80$	= 21,120	
	$DL = 440 \times 70$	= 30,800	
	Column	= 560	
		52,480#	200,070#, 7th-story column

7th floor—LL	$= 440 \times 60 \times 0.70$	$= 18{,}480$	
DL	$= 440 \times 70$	$= 30{,}800$	
Column		$= 700$	
		49,980#	250,050#, 6th-story column

Loads for the remaining floors are found in the same manner. The live load of the fifth floor will be $440 \times 60 \times 0.50 = 13{,}200\#$, and this will be the live load for each of the four floors below. Figure 62 shows a method of tabulating these results.

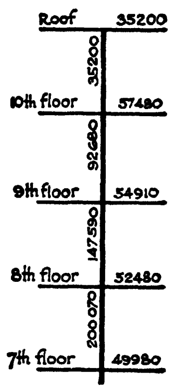


FIG. 62

82. **Column Base Plates.** Unless steel grillage is used, steel columns are supported at their bases by masonry, usually concrete. In order that the concrete be not crushed, a rolled steel slab or billet is attached to the foot of the column to distribute the column load over a larger area. It is essential that the base of the column and the steel slab be in absolute contact, and slabs 4″ and more in thickness are planed on the upper surface, the lower surface resting on a layer of cement grout on the concrete. The column is attached to the base plate by means of angle sections which are secured to the masonry footing by anchor bolts. Steel billets are also used as base plates for distributing column loads to grillage beams.

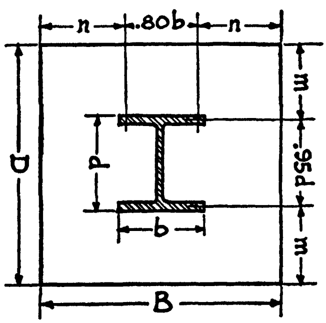


FIG. 63

83. **Design of Column Base Plates.** The area of the billet is readily found by dividing the column load by the allowable unit compressive stress of the concrete, usually 500, 625, or 750#/in^2, depending on its quality. The thickness of the billet is determined by assuming the plate to be an inverted cantilever with the maximum bending moment at the edge of the column and using the projection m or n, whichever is the greater. See Fig. 63. The design procedure follows.

STEP 1. Determine A, the required area of the plate.

$$A = \frac{P}{f_c}$$

in which A = the area of the plate, in square inches

P = the total column load, in pounds

f_c = the allowable compressive stress of the masonry, in pounds per square inch

$A = BD$ (see Fig. 63).

STEP 2. Make a diagram similar to Fig. 63, selecting a plate whose area is BD and whose dimensions m and n nearly equal.

STEP 3. Compute dimensions m and n using the values $0.95d$ and $0.80b$ as shown in Fig. 63.

STEP 4. Solve for t, the thickness of the plate in the following formula, using the value m or n, whichever is the greater.

$$t^2 = \frac{wm^2}{6{,}000} \qquad \text{or} \qquad t^2 = \frac{wn^2}{6{,}000}$$

in which t = the thickness of the plate, in inches

w = the actual bearing pressure on the masonry, in pounds per square inch $\left(w = \frac{P}{A}\right)$

m and n = the projection of the plate beyond the face of column, in inches.

The above formulas are based on an allowable extreme fiber stress of 18,000#/in^2. If the building code permits 20,000#/in^2, the denominators in the formulas should be 6,666 instead of 6,000.

EXAMPLE

EXAMPLE. Design a steel billet column base to support a 14″ WF 78# column having a column load of 300,000#. The plate will rest on a concrete pier having an allowable compressive stress of 500#/in^2 and the allowable extreme fiber stress of the steel billet is 18,000#/in^2.

SOLUTION. STEP 1. $A = \frac{P}{f_c} = \frac{300{,}000}{500} = 600\text{in}^2$, the required area.

Step 2. Select a plate 24″ x 26″. Actual $A = 624\text{in}^2$.

Step 3. $0.95d = 0.95 \times 14.06 = 13.3''$, $m = \dfrac{26 - 13.3}{2} = 6.35''$,

$$0.80b = 0.80 \times 12 = 9.6'', n = \frac{24 - 9.6}{2} = 7.2''.$$ Use this dimension.

Step 4. $w = \dfrac{P}{A} = \dfrac{300{,}000}{624} = 480\#/\text{in}^2$.

$$t^2 = \frac{wn^2}{6{,}000} = \frac{480 \times 7.2 \times 7.2}{6{,}000} = 4.14$$

$$t = 2.04''$$

Accept a plate 24″ x 26″ x $2\frac{1}{4}''$.

PROBLEM

83-A. A 12″ WF 53# column has an axial load of 240,000#. It is supported by a steel billet resting on a concrete footing for which the allowable compressive stress is $750\#/\text{in}^2$. Design the column base plate using an allowable extreme fiber stress of $20{,}000\#/\text{in}^2$.

84. **Grillage Foundations.** In order to distribute a column load over the foundation bed, steel grillage is occasionally used instead of reinforced concrete. This usually consists of two layers of beams as indicated in Fig. 65. The beams are encased in concrete not less than 4″ in thickness as a means of preservation, and the lower tier rests upon a bed of concrete to distribute the column load to the soil or rock foundation.

The principle involved in the design of the grillage is simply that of an inverted beam. The area of the footing is found by dividing the total load on the column by the allowable bearing value of the foundation bed. The dimensions of the footing are then determined, and this gives us l the length of the upper tier of beams. Let a be the length of the base plate and W the total load on the foundation. Now if we assume the point of maximum bending moment to be at the center of the beam, the upward force on either side of the section as well as the downward force is $\dfrac{W}{2}$, the

lever arms being respectively $\frac{l}{4}$ and $\frac{a}{4}$. See Fig. 64. Hence

$$M = \left(\frac{W}{2} \times \frac{l}{4}\right) - \left(\frac{W}{2} \times \frac{a}{4}\right) = \frac{Wl}{8} - \frac{Wa}{8} = \frac{W(l-a)}{8}$$

This quantity divided by n, the number of beams in the tier, gives us the maximum bending moment for each beam or $M = \frac{W(l-a)}{8 \times n}$.

Since $S = \frac{M}{f}$, $S = \frac{W(l-a)}{8 \times f \times n}$, the required section modulus for

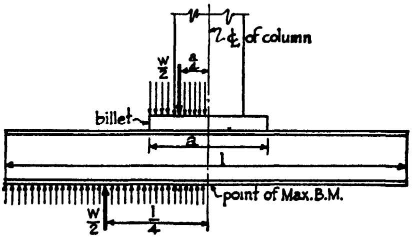


FIG. 64

each beam. The beam having been selected to resist flexure, it is then tested for shear and web buckling. The lower tier of beams is designed in a similar manner.

85. **Design of Grillage Beams.** In the formulas which follow, let

W = the total load on the foundation, in pounds
l = the length of the beam, in inches
a = the length of the loaded portion of the beam, in inches
f = the allowable stress of the beam in bending, in pounds per square inch
d = the depth of the beam, in inches
t = the thickness of the web of the beam, in inches
n = the number of beams in the tier
f_b = the web buckling resistance, in pounds per square inch
f_s = the web shearing resistance, in pounds per square inch

The design of grillage beams is facilitated by the following procedure.

STEP 1. Determine the area of the footing in square feet by dividing the column load W, by the allowable bearing value of the foundation bed.

Upper Tier

STEP 2. From the area of the footing determined in the preceding step determine l, the beam length of the upper tier, also n, the number of beams in the tier. The minimum clear distance between flanges should not exceed about 2″.

STEP 3. Bending. Compute S, the required section modulus of each beam, by the formula $S = \dfrac{W(l - a)}{8 \times f \times n}$, and select a trial beam.

STEP 4. Shearing. Compute f_s, the web shearing stress, by the formula $f_s = \dfrac{W(l - a)}{2 \times l \times n \times d \times t}$.

This value should not exceed 12,000#/in².

STEP 5. Buckling. Compute f_b, the web buckling resistance, by the formula $f_b = \dfrac{W}{n\left(a + \dfrac{d}{2}\right)t}$, not to exceed 15,000#/in².

STEP 6. If the shearing and buckling stresses fall within the permitted stresses, the beams selected in Step 3 are acceptable. If either of these stresses is too great, a larger beam is selected and tested again by Steps 4 and 5.

Lower Tier

The lower tier of beams is designed in the same manner as previously described for the upper tier.

EXAMPLE

EXAMPLE. The column load transmitted to a steel grillage foundation is 216,000#. Assuming the bearing value of the soil to be 6 tons per square foot, and the steel column base plate 19″ x 19″, design the steel grillage. See Fig. 65.

SOLUTION.

Upper Tier

STEP 1. $\dfrac{216{,}000}{12{,}000} = 18\text{ft}^2$, the area of the footing.

STEP 2. $18ft^2 = 4.5' \times 4'$. Accept a footing 48″ x 54″. Let 54″ = l, the length of the upper tier beams, and assume the number of beams to be 4. $n = 4$. See Fig. 65.

STEP 3. Bending.

$$S = \frac{W(l - a)}{8 \times f \times n} = \frac{216{,}000 \times (54 - 19)}{8 \times 18{,}000 \times 4} = 13.1\text{in}^3$$

the required section modulus of each of the 4 beams. As a trial beam select an 8″ I 18.4# (Table 5); note that d = 8″, b = 4″, and t = 0.27″.

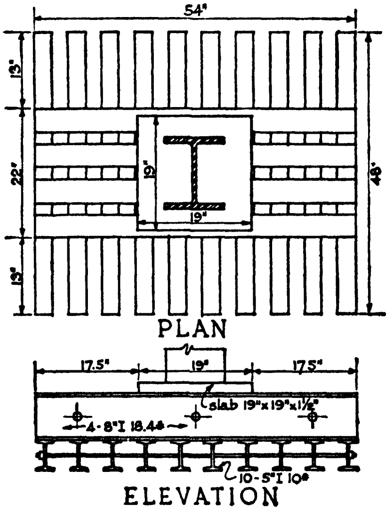


FIG. 65

STEP 4. Shearing.

$$f_s = \frac{W(l - a)}{2 \times l \times n \times d \times t} = \frac{216{,}000 \times (54 - 19)}{2 \times 54 \times 4 \times 8 \times 0.27} = 8{,}111\#/\text{in}^2$$

the web shearing stress; acceptable since it is less than $12{,}000\#/\text{in}^2$.

STEP 5. Buckling.

$$f_b = \frac{W}{n\left(a + \frac{d}{2}\right)t} = \frac{216{,}000}{4\left(19 + \frac{8}{2}\right)0.27} = 8{,}690\#/\text{in}^2$$

the web buckling resistance; acceptable, less than 15,000#/in^2.

STEP 6. Steps 4 and 5 show that four 8″ I 18.4# beams are acceptable for the upper tier.

Lower Tier

STEP 1. Not required for lower tier.

STEP 2. $l = 48''$, the length of the beams in the lower tier, and $a = 22''$ (see Fig. 65). Assume $n = 10$, the number of beams.

STEP 3. Bending.

$$S = \frac{W(l - a)}{8 \times f \times n} = \frac{216{,}000(48 - 22)}{8 \times 18{,}000 \times 10} = 3.9\text{in}^3$$

trial beams, ten 5″ I 10#. $d = 5''$, $b = 3''$ and $t = 0.21''$ (Table 5).

STEP 4. Shearing.

$$f_s = \frac{W(l - a)}{2 \times l \times n \times d \times t} = \frac{216{,}000 \times (48 - 22)}{2 \times 48 \times 10 \times 5 \times 0.21} = 5{,}571\#/\text{in}^2$$

Approved for shear.

STEP 5. Buckling.

$$f_b = \frac{W}{n\left(a + \frac{d}{2}\right)t} = \frac{216{,}000}{10\left(22 + \frac{5}{2}\right)0.21} = 4{,}198\#/\text{in}^2$$

Approved for buckling.

STEP 6. For the lower tier accept ten 5″ I 10#.

SECTION IV

REINFORCED CONCRETE

86. Concrete. Concrete is made by mixing cement, sand, and crushed stone together while dry and adding sufficient water to produce the desired consistency. After this plastic mixture is poured in forms, a chemical action takes place and the mass hardens, becoming what might be called an artificial stone. Concrete, though strong in compression, is relatively weak in resisting tensile and shearing stresses which develop in structural members. To overcome this lack of resistance, steel rods are placed in the concrete, and the result is reinforced concrete. In beams and slabs the principal function of the concrete is to resist compressive stresses while the steel rods resist tensile stresses.

87. Strength of Concrete. Various proportions of cement, aggregate and water produce concrete of different qualities. A mix commonly used for slabs, beams and columns is 1 part cement, 2 parts sand, and 4 parts pebbles or crushed stone with 7½ gallons of water for each 94# sack of cement. This proportion is written 1:2:4 or 1:6, and its assumed compressive strength at the end of 28 days is 2,000#/in^2. Since only a certain quantity of water can combine with the cement, an excess quantity of water reduces the strength and watertightness. Hence this water-cement ratio, the proportion of water to the quantity of cement, is an important item in controlling the strength of concrete. Whenever possible, various proportions of cement, aggregate, and water-cement ratio are tested to find a combination that will give concrete of the desired strength. When such preliminary tests are possible the allowable working stresses are given as percentages of the ultimate compressive strength developed at the 28-day period, f'_c. An abbreviated table of allowable stresses is given in Table 30.

The most recent codes permit an allowable extreme fiber stress in compression to be $0.45f'_c$, which for 2,000# concrete is 0.45 ×

2,000 or 900#/in^2. However, since $0.4f'_c$, 800#/in^2, is used in so many codes, it is used in many of the following examples.

Table 30. Allowable Stresses in Concrete

Description of Stress	Allowable Unit Stresses Pounds per Square Inch	
	For any strength of concrete as fixed by test in accordance with code	When strength of concrete is fixed by the water-cement ratio
	$n = \frac{30,000}{f'_c}$	$f'_c = 2,000\#$ $n = 15$
Flexure		
Extreme fiber stress in compression, f_c......	$0.40f'_c$	800
Shear		
Beams with no web reinforcement and without special anchorage of longitudinal steel	$0.02f'_c$	40
Beams with properly designed web reinforcement but without special anchorage of longitudinal steel....................	$0.06f'_c$	120
Bond		
In beams, slabs, and one-way footings:		
Plain bars..........................	$0.04f'_c$	80
Deformed bars.......................	$0.05f'_c$	100
Bearing		
When a concrete member has an area at least twice the area in bearing..............	$0.25f'_c$	500

88. **Steel Reinforcement.** The steel used in reinforced concrete is round or square rods or wire fabric. Rods or bars are either plain or deformed, the deformed bars being those with lugs or projections the purpose of which is to develop a greater bond between the concrete and steel. The grade of steel used for reinforcement is generally the "intermediate," and the allowable tensile stress is either 18,000#/in^2 or 20,000#/in^2, depending on the code specification. For web reinforcement the stresses are usually the same.

Table 31. Areas and Perimeters of Rods and Bars

Round Rods							
Size in Inches	¼	⅜	½	⅝	¾	⅞	1
Area, square inches...	0.05	0.11	0.20	0.31	0.44	0.60	0.79
Perimeter, inches.....	0.78	1.18	1.57	1.96	2.35	2.75	3.14
Square Bars							
Size in Inches	½	1	1⅛	1¼			
Area, square inches....	0.25	1.00	1.27	1.56			
Perimeter, inches......	2.00	4.00	4.50	5.00			

Table 31 gives the areas and perimeters of the rods and bars generally used for reinforcement, and the designer should confine himself to these sizes.

89. **Notation.** The most common type of reinforced concrete beam is designed on the assumption that the steel resists all the tensile stresses and the concrete the compressive stresses. This is

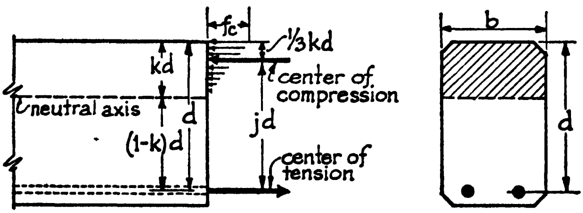


FIG. 66

shown graphically in Fig. 66, the concrete area resisting compression being the portion of the beam above the neutral surface indicated by cross hatching. Note particularly that d is the distance from the top of the beam to *the center line of the steel.* The following notation is used in the design formulas:

d = the effective depth of the beam, the distance from the uppermost surface in compression to the center of the longitudinal reinforcement, in inches

b = the width of a rectangular beam, in inches

k = the ratio of the depth of the neutral axis to the effective depth, d

j = the ratio between the distance from center of compression to the center of tension and the depth, d

f_c = the compressive unit stress in the extreme fiber of concrete, in pounds per square inch

f_s = the tensile unit stress in the steel, in pounds per square inch

A_s = the area of the cross section of tensile reinforcement, in square inches

p = the percentage of reinforcement or $p = \frac{A_s}{bd}$

n = the modulus of elasticity of steel divided by the modulus of elasticity of concrete or $\frac{E_s}{E_c}$

M = the bending moment, in inch-pounds

90. **Modulus of Elasticity of Concrete.** The modulus of elasticity of concrete E_c which is an indication of the stiffness of the material (Art. 17), depends on its strength. For concretes having an ultimate strength from 1,500#/in^2 to 2,200#/in^2 it is assumed to be about 2,000,000#/in^2, and E_s, the modulus of elasticity of steel, may be taken at 30,000,000#/in^2. Since $n = \frac{E_s}{E_c}$, $n = \frac{30{,}000{,}000}{2{,}000{,}000} = 15$. For stronger concretes (that is, those having ultimate strengths between 2,200#/in^2 and 2,900#/in^2) n is assumed to be 12. For concrete commonly used in building construction it is only necessary to remember that $n = 15$. All tables giving safe loads for reinforced concrete structural members indicate the unit stresses assumed and this value n. See Table 34.

91. **Design Formulas for Beams.** The following formulas may be used directly (with the aid of Table 32) in determining the depth and tensile reinforcement of rectangular beams and slabs:

Table 32. Formula Factors for Reinforced Concrete

$n = 15 \qquad (f'_c = 2{,}000\#/\text{in}^2)$

f_c	f_s	p	k	j	K $K = \frac{1}{2}f_cjk$
600	18,000 20,000	0.0056 0.0047	0.333 0.311	0.889 0.896	88.9 83.5
650	18,000 20,000	0.0063 0.0053	0.351 0.328	0.883 0.891	100.8 95.0
700	18,000 20,000	0.0072 0.0060	0.368 0.344	0.877 0.885	113.1 106.6
750	18,000 20,000	0.0080 0.0067	0.385 0.359	0.872 0.880	125.7 118.8
800	18,000 20,000	0.0089 0.0075	0.400 0.375	0.867 0.875	138.7 131.0
900	18,000 20,000	0.0107 0.0091	0.429 0.403	0.857 0.866	165.0 157.0

$$d = \sqrt{\frac{M}{Kb}} \tag{1}$$

$$A_s = pbd \tag{2}$$

$$A_s = \frac{M}{f_sjd} \tag{3}$$

$$f_c = \frac{2M}{jkbd^2} \tag{4}$$

$$f_s = \frac{M}{A_sjd} \tag{5}$$

in which d = the effective depth of the beam, in inches
b = the width of the beam, in inches
A_s = the area of the tensile steel reinforcement, in square inches

M = the maximum bending moment, in inch-pounds
K, p, j, and k are constants corresponding to specification unit stresses as given in Table 32.
f_s = the unit stress in tensile steel reinforcement, in pounds per square inch
f_c = the unit compressive stress in extreme fiber of concrete, in pounds per square inch.

The use of these fundamental formulas is illustrated in the following examples.

Formula (1), $d = \sqrt{\frac{M}{Kb}}$, is used to compute the effective depth of a rectangular beam after having assumed a width. The clear distance between supports should not exceed 32 times the width of the beam.

EXAMPLES

EXAMPLE. A beam has a load producing a maximum bending moment M equal to 94,500″#. Given the data f_s = 18,000#/in², f_c = 700#/in², and n = 15, compute the effective depth.

SOLUTION. Assume the breadth b to be 8″. Referring to Table 32, we find K = 113.1. Then

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{94{,}500}{113.1 \times 8}} = 10''$$

To find the *total* depth of the beam we add one-half the diameter of the steel reinforcement and 1½″ for fireproofing, or 10 + 0.5 + 1.5 = 12″. This gives us a beam 8″ x 12″, but remember the *effective* depth is 10″, the d found by the formula. See Fig. 67.

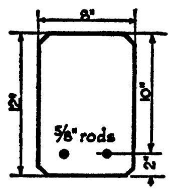


FIG. 67

Formula (2), $A_s = pbd$, determines the area of the tensile steel to produce balanced reinforcement and is used for rectangular beams and slabs. By balanced reinforcement is meant a beam whose cross-section areas of concrete and steel are proportioned so that each develops its full working stress simultaneously.

EXAMPLE. Determine the cross-section area of tensile reinforcement for the beam given in the preceding example.

SOLUTION. Data: $b = 8''$, $d = 10''$, and, from Table 32, $p = 0.0072$.

Then $A_s = pbd$ or $A_s = 0.0072 \times 8 \times 10 = 0.57\text{in}^2$, the total area of the tensile steel. Since the beam is 8″ in width, it is advisable to use 2 rods, and referring to Table 31 we find that two ⅝″ round rods are adequate.

Formula (3), $A_s = \dfrac{M}{f_s jd}$, is a general formula for finding the area of the steel reinforcement in beams. If it had been used in the previous example it would have given the same result. Suppose, however, that the breadth and effective depth of a beam are determined by certain building conditions and that the load is relatively small. It would be uneconomical to use the formula $A_s = pbd$, for this would result in using a steel area that would not be stressed up to its allowable unit stress. Formula (3), $A_s = \dfrac{M}{f_s jd}$, is applicable for this condition, and it is used also in a modified form for T-beams. See Art. 97.

EXAMPLE. A beam has a breadth of 8″, an effective depth of 10″, and is loaded to produce a maximum bending moment of 63,000″#. Data: $f_s = 18{,}000\#/\text{in}^2$, $f_c = 700\#/\text{in}^2$, and $n = 15$. What should be the area of the tensile steel reinforcement?

SOLUTION. Note that this problem is similar to the preceding one with the exception that the load is smaller. It was seen that, for a load producing a bending moment of 94,500″#, $A_s = 0.57\text{in}^2$, balanced reinforcement. Since in this example the bending moment is smaller, though the dimensions of the beam are similar, the steel area may be found by the formula $A_s = \dfrac{M}{f_s jd}$ or

$$A_s = \frac{63{,}000}{18{,}000 \times 0.877 \times 10} = 0.39\text{in}^2$$

Two ½″ round rods will supply this required area (Table 31).

Formulas (4) and (5), $f_c = \dfrac{2M}{jkbd^2}$ and $f_s = \dfrac{M}{A_s jd}$, are used for

checking the unit stresses in concrete and steel of beams previously designed.

PROBLEM

91-A. A rectangular reinforced concrete beam has a maximum bending moment of 1,000,000″#. Assuming the width to be 12″, determine the effective depth and the tensile reinforcement required. Specification data:

$$f_s = 18{,}000\#/\text{in}^2$$

$$f_c = 900\#/\text{in}^2$$

$$n = 15$$

92. **Diagonal Tension.** A concrete beam without tensile reinforcement will fail because of tensile stresses in the concrete as indicated in Fig. 68 (*a*). The area of the required tensile reinforcement is computed by means of formula (2) or (3) in Art. 91. If such reinforcement is introduced, the beam has a further tendency to fail by tensile stresses indicated in Fig. 68 (*b*). These cracks are due to a combination of tension and vertical shear, and the stress is known as *diagonal tension.* To prevent failure due to diagonal tension vertical steel rods called stirrups are used. For rectangular and T-beams they are illustrated in Figs. 68 (*c*) and 68 (*d*). They are generally round in section, ⅜″ or ½″ in diameter.

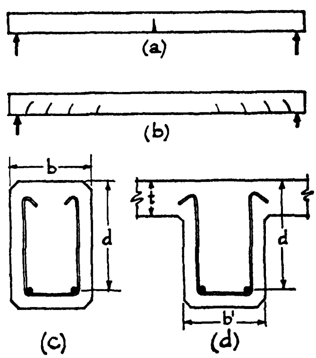


Fig. 68

Building codes generally permit 40#/in² for the unit shearing stress in concrete. If the load on the beam produces unit stresses exceeding this value it is necessary to use stirrups. This, known as web reinforcement, permits a unit stress of 120#/in². See Table 30.

The unit shearing stress, which is a measure of diagonal tension, is found by the formula

$$v = \frac{V}{jbd}$$

in which v = the unit shearing stress, in pounds per square inch

V = the total maximum vertical shear, in pounds (For uniformly loaded beams $V = \frac{W}{2}$, Art. 27.)

j = 0.875 or $\frac{7}{8}$, the value employed in computing shear

b = the width of the rectangular beam, in inches

d = the effective depth of the beam, in inches.

Since $j = \frac{7}{8}$,

$$v = \frac{V}{jbd} \quad \text{or} \quad v = \frac{8V}{7bd} \tag{6}$$

It is unnecessary to use web reinforcement throughout the entire length of a beam since the shearing stresses are maximum at the supports and decrease toward the center. To find the distance from the support in which web reinforcement is required we use the formula

$$a = \frac{L}{2} \times \frac{v'}{v} \tag{7}$$

in which a = the distance from the support in which stirrups are required, in feet

L = the span of the beam, in feet

v_c = the allowable shearing unit stress permitted in plain concrete, 40#/in^2 for 2,000# concrete

v = the maximum shearing unit stress in pounds per square inch, found from formula (6)

$v' = v - v_c$ = the excess shear to be resisted by the stirrups.

The distance in which web reinforcement is required having been found, it is next necessary to determine the spacing of the stirrups. This is found by the formula

$$s = \frac{A_v f_v}{v'b} \tag{8}$$

in which s = the spacing of the stirrups, in inches

A_v = the total cross-section area of the legs of one stirrup, in square inches

f_v = the allowable tensile unit stress in stirrups, in pounds per square inch, usually 18,000 or 20,000#/in^2

v' and b are as noted above.

A $\frac{3}{8}''$ round rod has an area of 0.11in^2 and, since a U-shaped stirrup has two legs, A_v for $\frac{3}{8}''$ stirrups $= 2 \times 0.11 = 0.22\text{in}^2$. Similarly A_v for $\frac{1}{2}''$ stirrups $= 2 \times 0.2 = 0.4\text{in}^2$.

The other factors in the formula are as noted above.

The first stirrup is placed $\frac{1}{2}s$ from the support. Where one-half the tensile reinforcement is raised for the negative bending moment, this steel is assumed to serve as web reinforcement for a distance of $\frac{3}{4}d$. If stirrups are required beyond this point, that is, toward the center of the beam, they are placed at intervals not exceeding $\frac{3}{4}d$. The use of formulas (6), (7), and (8) is illustrated in the example in Art. 93.

93. **Bond.** Computations for reinforced concrete are based on the assumption that there is perfect adhesion between the steel and the concrete. To increase the anchorage, deformed bars are used or the ends are formed into hooks having radii of not less than 3 or more than 6 bar diameters. The unit bond stress is computed by the formula

$$u = \frac{V}{\Sigma_0 jd} \tag{9}$$

in which u = the unit bond stress, in pounds per square inch

V = the total maximum vertical shear, in pounds

Σ_0 = the sum of the perimeters of all the horizontal tensile reinforcing rods, in inches (Table 31)

j = 0.875 or $\frac{7}{8}$, the value used in connection with shear computations

d = the effective depth of the beam, in inches.

For plain bars the bond stress should not exceed $80\#/\text{in}^2$, and deformed bars should not have unit stresses exceeding $100\#/\text{in}^2$. See Table 30.

EXAMPLE

EXAMPLE. A reinforced concrete beam, 22′ in length, between faces of supports has a uniformly distributed load of 22,000# exclusive of the weight of the beam. The ends of the beam are fully continuous. With the following data design the beam.

Data.

$$n = 15$$

$$f_s = 18{,}000\#/\text{in}^2$$

$$f_c = 800\#/\text{in}^2$$

$$f_v = 18{,}000\#/\text{in}^2$$

$$v_c = \text{limited to } 40\#/\text{in}^2$$

$$v = \text{limited to } 120\#/\text{in}^2$$

$$u = \text{limited to } 80\#/\text{in}^2 \text{ for plain bars}$$

SOLUTION.

Weight of Beam

First compute the approximate weight of the beam. Estimating the effective depth to be 1¼″ for each foot of span and adding 2″ for fireproofing, we find that the estimated total depth = (22 × 1¼) + 2 = 29½″. Call it 30″, and assume the width to be 10″.

Concrete weighs approximately 150#/ft^3. Then the estimated weight of the beam $= \dfrac{10 \times 30}{144} \times 22 \times 150 = 6{,}875\#$.

W, the total uniformly distributed load = 22,000 + 6,875 = 28,875#.

Bending Moment

The maximum bending moment $= \dfrac{Wl}{12} = \dfrac{28{,}875 \times 22 \times 12}{12} =$ 635,250″#. The reason for using the value $M = \dfrac{Wl}{12}$ is given in Art. 95.

Effective Depth

Formula (1), $d = \sqrt{\dfrac{M}{Kb}}$. Table 32 gives $K = 138.7$. Therefore $d = \sqrt{\dfrac{635{,}250}{138.7 \times 10}} = 21.5''$, the effective depth. Allowing ½″ for ½ the diameter of the tensile reinforcement, and 1½″ for fire-

proofing, the total depth of the beam equals $21.5 + 0.5 + 1.5 = 23.5''$.

Area of Steel

The formula for balanced reinforcement applies. Therefore

$$\text{Formula (2)}: \quad A_s = pbd, \ p = 0.0089 \quad \text{(Table 32)}$$

Then

$$A_s = 0.0089 \times 10 \times 21.5 = 1.91\text{in}^2$$

Since the beam is 10″ in width, use 4 rods. Table 31 gives the area of a ⅞″ round rod to be 0.6in^2; hence four ⅞″ round rods are acceptable for the tensile reinforcement: $4 \times 0.6 = 2.4\text{in}^2$, slightly more than required.

Stirrups

The total load W equals 28,875#; hence

$$\text{Maximum shear } V = \frac{28{,}875}{2} = 14{,}438\# \quad \text{(See Art. 27)}$$

$$\text{Formula (6)}: \quad v = \frac{V}{jbd} = \frac{8V}{7bd}$$

$$v = \frac{8 \times 14{,}438}{7 \times 10 \times 21.5} = 77\#/\text{in}^2, \text{ the shearing unit stress}$$

Since this stress exceeds $40\#/\text{in}^2$, it will be necessary to use stirrups to aid in resisting the diagonal tension.

To find the distance from the supports in which stirrups are required use formula (7):

$$a = \frac{L}{2} \times \frac{v'}{v} = \frac{22}{2} \times \left(\frac{77 - 40}{77}\right) = 5.2' = 5'\,3''$$

The spacing of stirrups is determined by the use of formula (8): $s = \dfrac{A_v f_v}{v'b}$. The sectional area of the two legs of a stirrup made up of a ⅜″ round rod is $2 \times 0.11 = 0.22\text{in}^2$; therefore

$$s = \frac{0.22 \times 18{,}000}{(77 - 40) \times 10} = 11''$$

Bond

The unit bond stress is found by formula (9): $u = \dfrac{V}{\Sigma_0 jd}$.

The perimeter of one ⅞″ round rod equals 2.75″; then

$$u = \frac{14{,}438}{(4 \times 2.75) \times 0.875 \times 21.5} = 71\#/\text{in}^2$$

This bonding unit stress being less than 80#/in², plain bars are acceptable.

Diagram

The beam indicating the position of the reinforcement is shown in Fig. 69. At the center of the span there are four ⅞″ round rods. Two of these are straight and continue to the center line of the columns; the other two are double-bend rods, raised at the ⅕

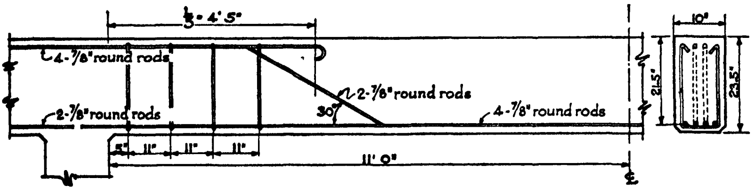


FIG. 69

point of the span, the inflection point. (See Art. 32.) The double-bend rods are lapped across the supports to the ⅕ point of the adjacent span where the ends are hooked. Equation (7) above showed that web reinforcement is required for a distance of 5′ 3″ beyond the support. In this instance only four stirrups are needed at each end of the beam since the bent-up rod serves as web reinforcement beyond the last stirrup.

94. Span Length. In computations for reinforced concrete, the length of freely supported beams and slabs is generally taken as the distance between centers of supports but should not exceed the clear span plus the depth of beam or slab. The span length for continuous or restrained beams is taken as the clear distance between faces of supports.

95. Bending Moments. For a simple beam, that is, a single span having no restraint at the supports, the maximum bending moment for a uniformly distributed load is at the center of the span, its magnitude being $M = \frac{Wl}{8}$, Fig. 70 (*c*). (See Art. 36.) This

condition may occur in practice, but reinforced concrete floor systems are poured simultaneously for several spans. This continuity results in what is known as "continuous beams." Figure 70 (*a*) shows exaggerated deflections of such a condition. From an inspection of this diagram one may readily see that positive bending moments occur between the supports and negative bending moments are developed over the supports. (See Art. 32.) The exact values of the maximum moments depend on several factors,

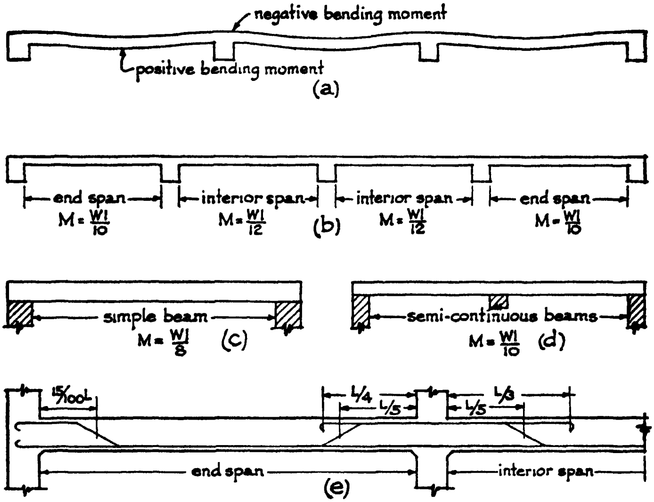


FIG. 70

but for uniformly distributed loads it is sufficient to accept the value of $\frac{Wl}{8}$ for simple spans, $\frac{Wl}{10}$ for end spans, and $\frac{Wl}{12}$ for interior spans. The latter two conditions are frequently called semi-continuous and fully continuous spans. The positive and negative moments are not necessarily equal for a given span, but this fact is ignored in most computations, an equal area of tensile steel reinforcement being used at the points of both positive and negative bending moments. For continuous beams and slabs of equal spans, uniformly loaded, it is customary to assume the inflection point to be at ⅕ the span length from the support. At this point the tensile steel reinforcement is raised from the lower to the upper

part of the beam to provide for the negative bending moment. See Fig. 70 (*e*).

96. **Design of Reinforced Concrete Beams.** The following steps may be used in the design of rectangular beams and slabs.

STEP 1. Estimate the approximate size and weight of the beam. It is customary to assume a width and to allow 1¼″ of depth for each foot of span; 150#/ft^3 may be used as the weight of concrete.

STEP 2. Compute the effective depth of the beam by formula (1), Art. 91:

$$d = \sqrt{\frac{M}{Kb}} \tag{1}$$

STEP 3. Compute the area of the tensile reinforcement by formula (2) or (3), Art. 91:

$$A_s = pbd \tag{2}$$

$$A_s = \frac{M}{f_s jd} \tag{3}$$

STEP 4. Compute v, the unit shearing stress, by formula (6), Art. 92:

$$v = \frac{V}{jbd} \tag{6}$$

If v exceeds 40#/in^2, stirrups are required.

STEP 5. Determine by formula (7), Art. 92, the distance from the support in which stirrups are necessary:

$$a = \frac{L}{2} \times \frac{v'}{v} \tag{7}$$

STEP 6. Compute s, the spacing of the stirrups, by formula (8), Art. 92:

$$s = \frac{A_v f_v}{v'b} \tag{8}$$

STEP 7. Determine the bond stress by formula (9), Art. 93:

$$u = \frac{V}{\Sigma_0 jd} \tag{9}$$

Unless special anchorage is provided, 80#/in^2 is generally the limit for plain bars and 100#/in^2 for deformed bars.

STEP 8. Make a diagram showing dimensions and position of steel reinforcement.

EXAMPLE

EXAMPLE. A fully continuous reinforced concrete beam has a span of 28′ between faces of supports and a uniformly distributed load of 32,000#, not including the weight of the beam. Design the beam with the following specifications:

$$f_s = 18{,}000\#/\text{in}^2$$

$$f_c = 800\#/\text{in}^2$$

$$f_v = 18{,}000\#/\text{in}^2$$

$$v_c = \text{limited to } 40\#/\text{in}^2$$

$$u = \text{limited to } 80\#/\text{in}^2 \text{ for plain bars}$$

$$n = 15$$

SOLUTION. STEP 1. Weight of beam. Assume 12″ to be the width of the beam. The approximate effective depth = 1.25 × 28 = 35″. If 2″ is allowed for fireproofing, 35 + 2 = 37″, the total estimated depth of beam.

The estimated weight of the beam $= \dfrac{12 \times 37}{144} \times 28 \times 150 =$ 12,950#.

Total load, $W = 32{,}000 + 12{,}950 = 44{,}950\#$; call it 45,000#.

STEP 2. Depth of beam. Since the beam is fully continuous, $M = \dfrac{Wl}{12}$ (Art. 95).

$$M = \frac{Wl}{12} = \frac{45{,}000 \times 28 \times 12}{12} = 1{,}260{,}000''\#$$

$$K = 138.7 \quad \text{(Table 32)}$$

Then

$$d = \sqrt{\frac{M}{Kb}} \qquad (1) \quad \text{Art. 91}$$

$$d = \sqrt{\frac{1{,}260{,}000}{138.7 \times 12}} = 27.5'', \text{ the effective depth}$$

If ½″ is allowed for ½ the diameter of the tensile reinforcement and 1½″ for fire- or dampproofing, 27.5 + 0.5 + 1.5 = 29.5″, the total depth of the beam.

STEP 3. Area of tensile reinforcement. From Table 32, $p = 0.0089$.

$$A_s = pbd \qquad (2) \quad \text{Art. 91}$$

$$A_s = 0.0089 \times 12 \times 27.5 = 2.94\text{in}^2$$

A beam 12″ in width will take four reinforcing rods, and since one 1″ round rod has an area of 0.79in² (Table 31) four 1″ round rods provide an area of 3.16in². Accept four 1″ rods.

STEP 4. Shearing unit stress. Since the total load $W = 45{,}000\#$, each reaction and hence the maximum shear $V = \frac{45{,}000}{2} = 22{,}500\#$.

In computations for shear, $j = 0.875$ or ⅞. Then

$$v = \frac{V}{jbd} \qquad (6) \quad \text{Art. 92}$$

$$v = \frac{22{,}500}{0.875 \times 12 \times 27.5} = 77\#/\text{in}^2$$

Since the shearing unit stress 40#/in², permitted in the concrete, is exceeded, web reinforcement is required.

STEP 5. Length of beam requiring stirrups. In accordance with data, $v_c = 40\#/\text{in}^2$.

$$a = \frac{L}{2} \times \frac{v'}{v} \qquad (7) \quad \text{Art. 92}$$

$$a = \frac{28}{2} \times \left(\frac{77 - 40}{77}\right) = 6.8' = 6'\,9''$$

the distance from the support in which stirrups are to be used.

STEP 6. Spacing of stirrups. If the diameter of the stirrup rod is assumed to be ½″, the cross-section area of the two legs = 2 × 0.2 = 0.4in² (Table 31).

$$s = \frac{A_v f_v}{v'b} \qquad (8) \quad \text{Art. 92}$$

$$s = \frac{0.4 \times 18{,}000}{(77 - 40) \times 12} = 15'', \text{ the spacing of the stirrups}$$

STEP 7. Bond. The perimeter of one 1″ round rod = 3.14″. Therefore Σ_0, the sum of the perimeters of the four tensile reinforcing rods, $= 4 \times 3.14 = 12.56''$, and $j = 0.875$. To find the bond stress,

$$u = \frac{V}{\Sigma_0 jd} \qquad \text{(9) Art. 93}$$

$$u = \frac{22{,}500}{(4 \times 3.14) \times 0.875 \times 27.5} = 75\#/\text{in}^2$$

Since this value does not exceed 80#/in², plain bars are acceptable.

STEP 8. Figure 71 shows the completed design with the steel reinforcement. Two double-bend bars are carried over the sup-

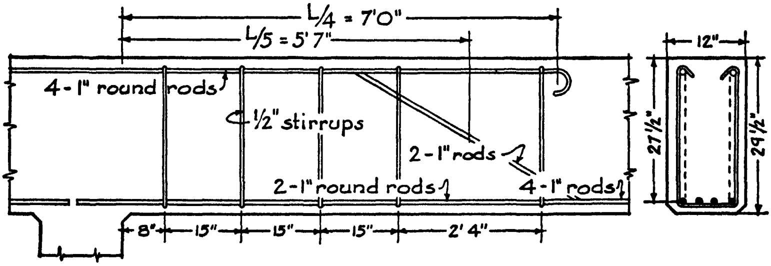


FIG. 71

port from the span on each side, making four rods to resist the negative bending moment. The two straight rods are carried to the center line of the support.

PROBLEM

96-A. A fully continuous, rectangular, reinforced concrete beam, 12″ in width, has a span of 20′ 0″ and a uniformly distributed load of 1,400 pounds per linear foot exclusive of the weight of the beam. Design the beam in accordance with the following specifications:

$f_s = 20{,}000\#/\text{in}^2$

$f_v = 20{,}000\#/\text{in}^2$

$f_c = 900\#/\text{in}^2$

$v_c =$ limited to $40\#/\text{in}^2$

$u =$ limited to $80\#/\text{in}^2$ for plain bars

$n = 15$

97. Design of T-Beams. When a floor system composed of slab and beams is designed, the slabs are designed first. This determines the thickness of the slab, and the slab acts as a compression flange in designing the beam. See Fig. 68 (*d*). The design of a T-beam is somewhat similar to that of a rectangular beam, two of the main differences being that the effective depth is determined by shear, and the tensile steel reinforcement is designed by a formula involving the slab thickness. Design procedure:

STEP 1. Assume b', the width of the web; estimate the approximate size and weight of the beam, allowing 1″ of depth for each foot of span, t, the slab thickness, having been previously determined.

STEP 2. Compute d by the formula $d = \frac{V}{vjb'}$. Since, when web reinforcement is used, v is limited to 120#/in² by most building codes and $j = 0.875$ for shear computations,

$$d = \frac{V}{vjb'} \quad \text{or} \quad d = \frac{V}{105b'} \tag{10}$$

in which d = the effective depth of the beam, the depth from top of slab to center of tensile reinforcement, in inches

b' = the width of the web of a T-beam, in inches [See Fig. 68 (*d*).]

V = the total maximum vertical shear, in pounds (For uniformly loaded beams $V = \frac{W}{2}$)

v = the allowable shearing unit stress when web reinforcement is used, in pounds per square inch

j = 0.875 or ⅞, the value used in shear computations.

STEP 3. Compute the area of tensile steel reinforcement by the formula

$$A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)} \tag{11}$$

in which A_s = the area of tensile steel reinforcement, in square inches

M = the maximum bending moment, in inch-pounds

f_s = the allowable tensile unit stress in steel reinforcement, in pounds per square inch

d = the effective depth of the beam, in inches

t = the thickness of the slab, in inches.

STEP 4. Compute the distance from the support in which web reinforcement is required.

$$a = \frac{L}{2} \times \frac{v'}{v} \qquad (7) \quad \text{Art. 92}$$

When this formula is used for T-beams, v_c is the allowable shearing unit stress of plain concrete, generally 40#/in^2, and v is 120#/in^2 or 150#/in^2, the value used in formula (10).

STEP 5. Determine the spacing of the stirrups by the formula

$$s = \frac{A_v f_v}{v' b'} \qquad (8) \quad \text{Art. 92}$$

Note that this formula is similar to that used for rectangular beams with the exception that b' is used instead of b, b' being the width of the web of a T-beam.

STEP 6. Compute the bond stress

$$u = \frac{V}{\Sigma_0 jd} \qquad (9) \quad \text{Art. 93}$$

STEP 7. Prepare a diagram showing dimensions and sizes of reinforcement.

EXAMPLE

EXAMPLE. Design a T-beam having a span 18′ between faces of supports, a load of 26,000# exclusive of the weight of the beam, the ends fully continuous and the slab 4″ in thickness.

Data:

$$n = 15$$

$$f_s = 20{,}000\#/\text{in}^2$$

$$f_c = 800\#/\text{in}^2$$

$$f_v = 20{,}000\#/\text{in}^2$$

$$v_c = \text{limited to } 40\#/\text{in}^2$$

$$v = \text{limited to } 120\#/\text{in}^2$$

SOLUTION. STEP 1. Assume $b' = 10''$. The estimated depth equals $18 + 2 = 20''$, $2''$ being the allowance for fireproofing.

$$\text{Estimated weight of beam} = \frac{10 \times 20}{144} \times 18 \times 150 = 3{,}750\#.$$

Total weight $W = 26{,}000 + 3{,}750 = 29{,}750\#$.

STEP 2.

$$V = \frac{29{,}750}{2} = 14{,}875\#$$

$$d = \frac{V}{vjb'} = \frac{14{,}875}{120 \times 0.875 \times 10} = 14.1''$$

Accept a total depth of 16″.

STEP 3.

$$M = \frac{Wl}{12} = \frac{29{,}750 \times 18 \times 12}{12} = 535{,}500''\#$$

$$A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)} = \frac{535{,}500}{20{,}000 \times \left(14.1 - \frac{4}{2}\right)} = 2.21\text{in}^2$$

the area of the tensile steel reinforcement. Four ⅞″ round rods provide an area of 2.4in^2 (Table 31). Accept four ⅞″ round rods.

STEP 4.

$$a = \frac{L}{2} \times \frac{v'}{v} = \frac{18}{2} \times \frac{120 - 40}{120} = 6'$$

the distance from the support in which stirrups are required.

STEP 5. The area of the two legs of a ½″ stirrup, $A_v = 2 \times 0.2 = 0.4\text{in}^2$ (Table 31).

$$s = \frac{A_v f_v}{v'b'}$$

$$s = \frac{0.4 \times 20{,}000}{(120 - 40) \times 10} = 10'', \text{ the spacing of the stirrups}$$

STEP 6. The perimeter of one ⅞″ round rod = 2.75″.

$$u = \frac{V}{\Sigma_0 jd} = \frac{14{,}875}{(4 \times 2.75) \times 0.875 \times 14.1} = 110\#/\text{in}^2$$

This stress being in excess of $100\#/\text{in}^2$, it will be necessary to use deformed bars with hooks.

STEP 7. The dimensions of the beam are shown in Fig. 72.

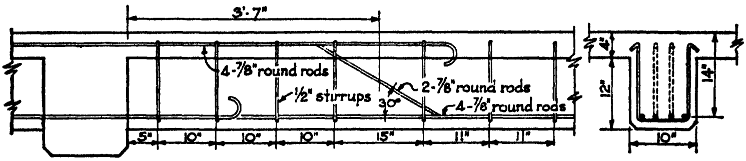


FIG. 72

PROBLEM

97-A. A T-beam has a span of 20′ 0″ between faces of supports and a load of 2,000 pounds per linear foot, not including the weight of the beam. The beam is fully continuous and the slab has a thickness of 6″. Assume the width of the stem to be 10″ and design the beam in accordance with the following specification data:

$$n = 15$$

$$f_s = 18{,}000\#/\text{in}^2$$

$$f_c = 800\#/\text{in}^2$$

$$f_v = 18{,}000\#/\text{in}^2$$

$$v_c = 40\#/\text{in}^2$$

$$v = 120\#/\text{in}^2$$

$$u = \text{limited to } 80\#/\text{in}^2 \text{ for plain bars}$$

98. Design of Floor Slab. One-Way Reinforcement. One of the commonly used reinforced concrete floor systems is the slab with one-way reinforcement. The slab frames into beams, the beams into girders, and the girders transfer their loads to the columns. To design the slab a strip 12″ in width is considered, and the computations are similar to a beam uniformly loaded. The depth and steel reinforcement required for the 12″ strip are of

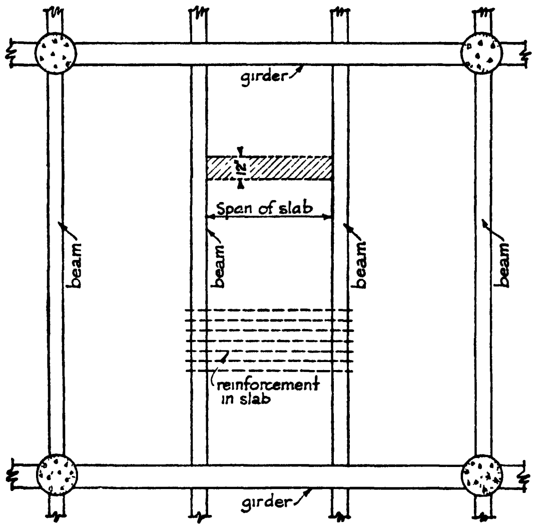


FIG. 73

course used for the entire slab, the reinforcement running parallel to the imaginary strip, that is, from beam to beam. See Fig. 73. In addition to the tensile reinforcement, $\frac{3}{8}$″ rods spaced approximately 18″ on centers are run parallel to the beams. These are known as temperature rods, their purpose being to provide against the destructive effect of changes in temperature and shrinkage. The actual unit shearing stress v in slabs is generally quite low and web reinforcement is unnecessary, but with this exception the design steps for beams outlined in Art. 96 may be followed.

Table 33, giving rod diameters and spacing, is convenient in slab computations. The tensile steel reinforcement is found by means of formula (2) or (3), Art. 91. This, of course, is the required area for a strip 12″ in width, and the table shows at a

glance the various combinations of rod diameter and spacing that may be used.

Table 33. Areas of Rods and Bars in Reinforced Concrete Slabs per Foot of Width

	Areas of Round Rods in Inches							Areas of Square Bar in Inches			
Spacing, In	1/4"	3/8"	1/2"	5/8"	3/4"	7/8"	1"	1/2"	1"	1 1/8"	1 1/4"
3	0.20	0.44	0.79	1.23	1.77	2.41	3.14	1.00	4.00	5.06	6.24
3 1/2	0.17	0.38	0.67	1.05	1.51	2.06	2.69	0.86	3.43	4.34	5.36
4	0.15	0.33	0.59	0.92	1.33	1.80	2.36	0.75	3.00	3.80	4.68
4 1/2	0.13	0.29	0.52	0.82	1.18	1.60	2.09	0.67	2.67	3.37	4.16
5	0.12	0.26	0.47	0.74	1.06	1.44	1.88	0.60	2.40	3.04	3.75
5 1/2	0.11	0.24	0.43	0.67	0.96	1.31	1.71	0.55	2.18	2.76	3.41
6	0.10	0.22	0.39	0.61	0.88	1.20	1.57	0.50	2.00	2.53	3.12
6 1/2	0.09	0.20	0.36	0.57	0.82	1.11	1.45	0.46	1.85	2.33	2.88
7	0.08	0.19	0.34	0.53	0.76	1.03	1.35	0.43	1.72	2.17	2.68
7 1/2	0.08	0.18	0.31	0.49	0.71	0.96	1.26	0.40	1.60	2.02	2.50
8	0.07	0.17	0.29	0.46	0.66	0.90	1.18	0.38	1.50	1.90	2.34
8 1/2	0.07	0.16	0.28	0.43	0.62	0.85	1.11	0.35	1.41	1.78	2.20
9	0.07	0.15	0.26	0.41	0.59	0.80	1.05	0.33	1.33	1.69	2.08
9 1/2	0.06	0.14	0.25	0.39	0.56	0.76	0.99	0.32	1.26	1.60	1.97
10	0.06	0.13	0.24	0.37	0.53	0.72	0.94	0.30	1.20	1.52	1.87
11	0.05	0.12	0.21	0.33	0.48	0.66	0.86	0.27	1.09	1.38	1.70
12	0.05	0.11	0.19	0.31	0.44	0.60	0.78	0.25	1.00	1.27	1.56

EXAMPLE

EXAMPLE. Design a reinforced concrete slab with one-way reinforcement for a fully continuous slab having a panel width between faces of beams of 11′ 0″. The live load is 70#/ft^2, floor finish 20#/ft^2 and a suspended plaster ceiling weighs 10#/ft^2.

Specification data:

$$n = 15$$

$$f_s = 20{,}000\#/\text{in}^2$$

$$f_c = 800\#/\text{in}^2$$

$$v_c = \text{limited to } 40\#/\text{in}^2$$

SOLUTION. The steps enumerated in Art. 96 are followed.

STEP 1. If the depth of the slab is assumed to be ½″ for each foot of span, ½ × 11 = 5½″. At 150#/ft³, the weight of the slab is approximately 70#/ft².

Live load	=	70
Floor finish	=	20
Plastered ceiling	=	10
Slab	=	70
Total weight	=	170#/ft²

$W = 11 \times 170 = 1{,}870\#$, the total weight for a strip 12″ in width.

STEP 2.

$$M = \frac{Wl}{12} = \frac{1{,}870 \times 11 \times 12}{12} = 20{,}570''\#$$

$$K = 131.0 \quad \text{(Table 32)}$$

$$b = 12'', \text{ the imaginary strip 12'' wide}$$

$$d = \sqrt{\frac{M}{Kb}} \qquad \text{Formula (1) Art. 91}$$

$$d = \sqrt{\frac{20{,}570}{131 \times 12}} = 3.5'', \text{ the effective depth}$$

Total depth of slab = 3.5 + 0.25 + 0.75 = 4½″. In this equation, 0.25 equals ½ the assumed diameter of the reinforcing rods, and 0.75 is the depth of concrete below the reinforcement for fireproofing.

STEP 3. From Table 32, $p = 0.0075$.

$$A_s = pbd \qquad \text{(2) Art. 91}$$

$A_s = 0.0075 \times 12 \times 3.5 = 0.315\text{in}^2$, the area of steel reinforcing rods for each 12″ of width. Referring to Table 33 we find that ½″ round rods spaced 7½″ on centers provide an equivalent area of 0.31in² for each 12″ of width. Now ⅜″ rods would require 4″ spacing and ⅝″ rods 12″ spacing. The maximum spacing of rods is usually 3 times the effective depth. Accept for the present ½″ round rods 7½″ spacing.

STEP 4.

$$V = \frac{W}{2} = \frac{1{,}870}{2} = 935\#$$

$j = 7/8$ or 0.875, the value used in shear computations

$$v = \frac{V}{jbd} \qquad (6) \quad \text{Art. 92}$$

$$v = \frac{935}{0.875 \times 12 \times 3.5} = 25\#/\text{in}^2, \text{ the unit shearing stress}$$

Since this value is less than 40#/in², web reinforcement is unnecessary. In slabs, stirrups are impracticable, and if v is excessive a greater depth of slab must be used.

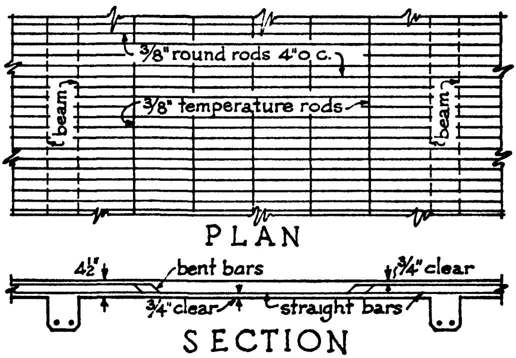


FIG. 74

STEPS 5 and 6. These steps do not apply when web reinforcement is omitted.

STEP 7. Since the tensile reinforcement consists of ½″ round rods spaced 7½″ on centers, 12 ÷ 7.5 = 1.6 the equivalent number of rods for each 12″ of width. A ½″ round rod has a perimeter of 1.57″ (Table 31).

$$u = \frac{V}{\Sigma_0 jd} \qquad (9) \quad \text{Art. 93}$$

$$u = \frac{935}{(1.6 \times 1.57) \times 0.875 \times 3.5} = 120\#/\text{in}^2$$

which is the bonding unit stress. Since it is high, try 3/8″ rods 4″ on center. Then

$$u = \frac{935}{(3 \times 1.18) \times 0.875 \times 3.5} = 86\#/\text{in}^2$$

Accept 3/8″ round deformed rods 4″ spacing.

STEP 8. See Fig. 74. The inflection point is taken at the 1/5 points of the span. The bends should not be made all at one place, but there should be a slight staggering to provide for variations in the inflection point due to eccentric loading. The alternate rods from each span are carried over the supporting beam, thereby furnishing adequate steel for the negative bending moment.

PROBLEM

98-A. Design a one-way fully continuous reinforced concrete slab for a span of 9′ 0″. The live load is 100#/ft^2, the floor finish is 20#/ft^2, and there is a plastered ceiling. Specification data:

$$f_s = 20{,}000\#/\text{in}^2$$

$$f_c = 800\#/\text{in}^2$$

$$v_c = \text{limited to } 40\#/\text{in}^2$$

$$u = \text{limited to } 80\#/\text{in}^2 \text{ for plain bars}$$

$$n = 15$$

99. Safe Load Table for One-Way Slabs. Table 34 gives safe superimposed loads for slabs with one-way reinforcement. Note the specification data in accordance with which this table has been computed. The first column gives the *total* thickness of slab and includes 3/4″ for fireproofing. By use of this table one may readily select the slab thickness and reinforcement for various loads and end conditions. In the previous example, for instance, the superimposed load is 70 + 20 + 10 = 100#/ft^2. Under the column "11′" for an interior span, that is ends fully continuous, we find 104#/ft^2 the superimposed load. Reading to the left we note that a 4½″ slab with ½″ round rods spaced 7½″ on centers will be adequate.

$f_s = 20,000\#/in^2$
$f_c = 800\#/in^2$
$n = 15$
¾″ clear fireproofing

Table 34. Concrete Slabs—One-Way Reinforcement *

Total Thickness of Slab	Required Steel Area	Reinforcing Steel Size	Reinforcing Steel Spacing	Condition of Continuity of Span	Safe Superimposed Load in Pounds per Square Foot — Span in feet																				
					4	4½	5	5½	6	6½	7	7½	8	8½	9	9½	10	10½	11	11½	12	12½	13	13½	14
In	In²	In	In																						
3	0 19	⅜φ	7	Simple	240	182	140	109	86	67	53	41													
				end	309	236	184	146	116	94	75	61	49	39											
				interior		292	229	183	146	120	98	81	66	54	44	36									
3½	0 23	⅜φ	5½	Simple		290	231	183	147	119	96	78	63	51	41										
				end			300	240	195	159	131	109	90	75	62	51	42	31							
				interior				297	242	200	167	139	117	99	83	70	59	49	41	34					
4	0 28	⅜φ	4½	Simple				275	223	182	150	125	103	86	71	59	48	39							
				end					292	241	201	168	142	120	102	86	73	56	47	39	32				
				interior						299	251	212	181	154	132	114	98	84	72	61	52	44	37		
4½	0 32	½φ	7½	Simple					301	249	206	172	145	122	103	87	73	61	50	41	33				
				end							272	230	196	167	143	122	105	84	71	60	51	42	35		
				interior								288	246	211	182	158	137	119	104	90	78	67	58	50	42
5	0 36	½φ	6½	Simple							280	235	199	170	144	123	105	89	76	64	54	44	36		
				end									265	227	196	169	147	119	103	89	77	65	56	47	39
				interior										286	249	216	189	165	145	127	112	98	86	75	65
5½	0 41	½φ	5½	Simple									264	226	194	167	144	124	107	92	79	67	57	48	39
				end											259	226	197	162	141	123	108	93	81	71	61
				interior													250	220	195	172	152	135	120	106	94
6	0 45	½φ	5	Simple										288	248	216	187	163	142	123	107	93	80	69	59
				end												288	253	210	185	162	143	126	111	97	85
				interior														282	250	223	198	177	158	141	126
6½	0 50	½φ	4½	Simple												271	237	207	181	159	139	122	107	93	81
				end														264	233	206	183	162	144	127	113
				interior																279	250	224	201	180	162
7	0 53	⅝φ	6½	Simple													282	247	217	192	168	148	131	115	101
				end															278	247	220	196	174	155	138
				interior																		267	240	216	195
7½	0 58	⅝φ	5½	Simple															266	235	208	185	163	145	128
				end																	268	239	214	191	171
				interior																				264	239

* Courtesy of the Portland Cement Association.

PROBLEMS

99-A. By the use of Table 34, design a one-way reinforced concrete slab for a span of 10′ 0″. The slab is fully continuous, the live load is 100#/ft^2, the floor finish is 30#/ft^2, plastered ceiling 10#/ft^2, and the allowance for movable partitions is 15#/ft^2.

99-B. Design a slab in accordance with the data given in Prob. 99-A with the exception that the span is 11′ 6″.

100. Design of Slabs with Two-Way Reinforcement. When floor panels are square or nearly so, two-way reinforcement may be used. If the slab length equals or exceeds 1½ times the width, the

Table 35. Load Distribution in Slabs with Two-Way Reinforcement

Ratio of Length to Width of Panel	Ratio of Load Carried by Shorter Span	Ratio of Load Carried by Longer Span
1.00	0.50	0.50
1.05	0.55	0.45
1.10	0.60	0.40
1.15	0.65	0.35
1.20	0.70	0.30
1.25	0.75	0.25
1.30	0.80	0.20
1.35	0.85	0.15
1.40	0.90	0.10
1.45	0.95	0.05
1.50	1.00	0.

load is carried by reinforcement in the short direction only. For uniformly distributed loads, Table 35 gives the proportion of load carried by the shorter and longer spans. This system of reinforcement consists of two bands of rods running at right angles to

each other. The bending moment is less in the portions of the band which are adjacent and parallel to the supporting beams, and the ¼ width of band on each side may have the moments reduced 50 per cent. The method of designing a slab with two-way reinforcement is similar to that in which the reinforcement runs in one direction only. However, since the rods in one direction are placed above those in the other, the effective depth is different for each system of rods.

EXAMPLE

Example. Design with two-way reinforcement a floor panel 16′ x 20′ having a live load of $70\#/ft^2$ and a finished floor load of $25\#/ft^2$. The end conditions are fully continuous.

Specification data:

$$f_s = 20{,}000\#/in^2$$

$$f_c = 800\#/in^2$$

$$n = 15$$

Solution. If the depth of slab is assumed to be 6″, the weight of slab equals $75\#/ft^2$. (One cubic foot of concrete weighs 150#.)

Live load	=	70
Finished floor	=	25
Slab	=	75
Total weight	=	$170\#/ft^2$

Ratio of length to width of panel $= \frac{20}{16} = 1.25$. Referring to Table 35 we find that 75 per cent of total load will be carried by the short span and 25 per cent by the long span. The loads carried on strips of slab 12″ in width are

$$0.75 \times 170 \times 16 = 2{,}040\#, \text{ total load carried by short span}$$

$$0.25 \times 170 \times 20 = 850\#, \text{ total load carried by long span}$$

Maximum bending moment for short span (Art. 95),

$$M = \frac{Wl}{12} = \frac{2{,}040 \times 16 \times 12}{12} = 32{,}640''\#$$

Maximum bending moment for long span,

$$M = \frac{Wl}{12} = \frac{850 \times 20 \times 12}{12} = 17{,}000''\#$$

$$d = \sqrt{\frac{M}{K \times b}} \qquad (1) \quad \text{Art. 91}$$

$$K = 131 \quad \text{(Table 32)}$$

$$\text{Short span, } d = \sqrt{\frac{32{,}640}{131 \times 12}} = 4.5''$$

$$\text{Long span, } d = \sqrt{\frac{17{,}000}{131 \times 12}} = 3.3''$$

The effective depth of the short span determines the slab thickness in this instance; therefore accept 6″ for the total depth of slab. If 1″ is allowed for fireproofing and ½″ for the assumed diameter of the reinforcing rods, $d = 6 - (1 + 0.25) = 4.75''$, the actual effective depth for short span. The rods for the long span will be placed above the reinforcement for the short span and

$d = 6 - (1 + 0.25 + 0.5) = 4.25''$, the actual d for the long span

$$A_s = \frac{M}{f_s jd} \qquad (3) \quad \text{Art. 91}$$

$$j = 0.875 \quad \text{(Table 32)}$$

$$\text{Short direction, } A_s = \frac{32{,}640}{20{,}000 \times 0.875 \times 4.75} = 0.39\text{in}^2$$

Use ½″ round-rods 6″ on centers (Table 33).

$$\text{Long direction, } A_s = \frac{17{,}000}{20{,}000 \times 0.875 \times 4.25} = 0.22\text{in}^2$$

Use ½″ round rods 11″ on centers (Table 33).

This reinforcement will be used in the central two quarters of the long and short spans, and since a reduction of 50 per cent of the bending moment is permitted in the quarter bands parallel and adjacent to the beams, use ½″ round rods 11″ on centers for the

outer bands of the short span and ½″ round rods 22″ on centers for the outer bands of the long span.

These are the theoretical spacings. The actual spacings depend on the dimensions of the panel and are shown in Fig. 75.

Alternate rods are bent up over the supporting beams to provide for the negative bending moments.

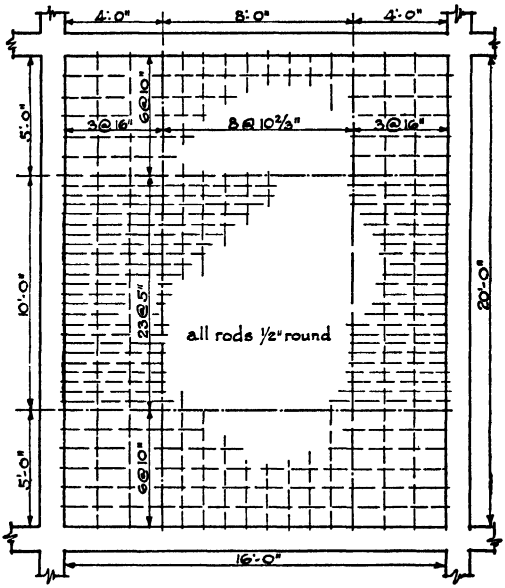


FIG. 75

$$\text{Bond stress } u = \frac{V}{\Sigma_0 jd} \qquad (9) \quad \text{Art. 93}$$

$$V = \frac{2,040}{2} = 1,020\#, \text{ the maximum shear for the short span}$$

$$\frac{12}{6} = 2 \text{ the number of rods at } 6'' \text{ spacing in each } 12'' \text{ strip}$$

$1.57'' =$ the perimeter of a ½″ round rod (Table 31)

$j = 0.875$ (Table 32)

$$u = \frac{1,020}{(2 \times 1.57) \times 0.875 \times 4.75} = 78\#/\text{in}^2$$

the bond stress for short span. Similarly,

$$u = \frac{425}{(1.09 \times 1.57) \times 0.875 \times 4.25} = 66\#/\text{in}^2$$

the bond stress for the long span. Plain rods are acceptable.

PROBLEM

100-A. A two-way reinforced concrete floor panel is 15′ 0″ square, the live load is $150\#/\text{ft}^2$, the total dead load is $110\#/\text{ft}^2$, and the end conditions are fully continuous. Design the slab in accordance with the following specifications.

$f_s = 20{,}000\#/\text{in}^2$

$f_c = 800\#/\text{in}^2$

$v_c =$ limited to $40\#/\text{in}^2$

$u =$ limited to $80\#/\text{in}^2$ for plain bars

$n = 15$

101. **Ribbed Slab with Metal Pans.** One of the most economical systems of floor construction for long spans with light loading consists of the use of metal pan fillers between concrete joists. The steel forms, often known as "tin pans," are generally 20″ in width. The ribs are usually 5″ wide at the bottom and are slightly wider at the top. The slab over the pans varies from 2″ to 3″ in thickness; it is cast integrally with the ribs, and since together they form a T-section, the computations for the design are similar to those for a T-beam. Ribbed floors with tile fillers are designed in a similar manner. See Fig. 76 (*b*).

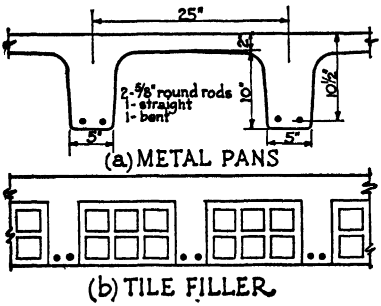


FIG. 76

EXAMPLE

EXAMPLE. Design a ribbed floor with metal pans for a span of 18′. The live load is $70\#/\text{ft}^2$, floor finish $25\#/\text{ft}^2$, and the end condition fully continuous.

Table 36. Weights of Ribbed Slabs in Pounds per Square Foot

Terra-Cotta Tile, 12″ x 12″

Depth of tile......	4″	6″	8″	10″	12″
2″ slab 4″ web					
Weight...........	50#	60#	70#	80#	90#
2½″ slab 4″ web					
Weight...........	56#	66#	76#	86#	96#

Metal Pans, 20″ wide

Depth of pan	4″	6″	8″	10″	12″
2″ slab 5″ web					
Weight...........	36#	42#	50#	57#	65#
2½″ slab 5″ web					
Weight...........	42#	48#	55#	63#	71#

Specification data:

$$n = 15$$

$$f_s = 20{,}000\#/\text{in}^2$$

$$f_c = 800\#/\text{in}^2$$

$$v_c = \text{limited to } 60\#/\text{in}^2.$$

SOLUTION. Assume a slab 2″ in depth and a pan 10″ deep. The pans will be 20″ wide, and a 5″ web results in ribs 25″ on centers. The depth of slab and pan determine d, the effective depth of the reinforcement, which is 10½″. See Fig. 76 (*a*)

The weight of this floor (Table 36) is 57#/ft².

Live load	=	70
Finished floor	=	25
Ribbed slab	=	57
Total load	=	152#/ft²

The total load on one rib 18′ in length, since ribs are 25″ on centers,

$$= 152 \times \frac{25}{12} \times 18 = 5{,}700\#.$$

$$M = \frac{Wl}{12} \qquad \text{(Art. 95)}$$

$$M = \frac{5{,}700 \times 18 \times 12}{12} = 102{,}600''\#$$

$$A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)} \qquad \text{(11) Art. 97}$$

$$A_s = \frac{102{,}600}{20{,}000\left(10.5 - \frac{2}{2}\right)} = 0.54\text{in}^2$$

Use two ⅝″ round rods in each web (Table 36).

$$f_c = \frac{2M}{bt\left(d - \frac{t}{2}\right)}$$

In this formula b = spacing of webs on centers, in inches.

$$f_c = \frac{2 \times 102{,}600}{25 \times 2 \times \left(10.5 - \frac{2}{2}\right)} = 432\#/\text{in}^2$$

the stress on extreme fiber of concrete; acceptable.

$$v = \frac{V}{jbd} \qquad (6) \quad \text{Art. 92}$$

$$v = \frac{\frac{5{,}700}{2}}{0.875 \times 6 \times 10.5} = 51\#/\text{in}^2$$

Acceptable, since $60\#/\text{in}^2$ is permitted for this type of construction. In this equation $b = 6''$, the average width of the web.

$$u = \frac{V}{\Sigma_0 jd} \qquad (9) \quad \text{Art. 93}$$

One $\frac{5}{8}''$ round rod has a perimeter of 1.96″ (Table 31).

$$u = \frac{\frac{5{,}700}{2}}{(2 \times 1.96) \times 0.875 \times 10.5} = 79\#/\text{in}^2$$

Use plain rods. Raise alternate rods at the $\frac{1}{5}$ points of span and carry to the $\frac{1}{5}$ point of the adjacent span. Straight rods to terminate at center line of girder.

PROBLEM

101-A. An interior panel of a floor is to be designed for a ribbed slab, metal-tile fillers 8″ in depth to be used. The clear span of the T-beam construction is 18′ 0″ and the end conditions are fully continuous. The live load is $50\#/\text{ft}^2$, the floor finish is $20\#/\text{ft}^2$, and there is a suspended ceiling of plaster on metal lath. Assuming the slab to be 2″ in thickness, design the ribbed slab in accordance with the following specification data:

$f_s = 20{,}000\#/\text{in}^2$

$f_c = 800\#/\text{in}^2$

$v_c =$ limited to $60\#/\text{in}^2$

$u =$ limited to $100\#/\text{in}^2$ for deformed bars

$n = 15$

102. Tied Columns. Most of the concrete columns used in building construction are known as short columns, and their length should not be greater than 10 times their least cross-section dimen-

sion. Their minimum thickness or diameter should not be less than 10″ and should have a minimum gross area of 120in^2, but posts not continuous from story to story may have a minimum diameter or thickness of 6″.

A tied column has reinforcement consisting of longitudinal bars and separate lateral ties. The ratio of the effective cross-section area of vertical reinforcement to the gross column area p_g should not be less than 0.01 or more than 0.04. The longitudinal reinforcement should consist of at least 4 bars of minimum diameter of ⅝″.

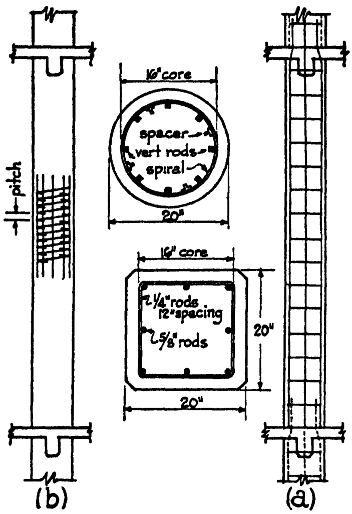


FIG. 77

Lateral ties should be at least ¼″ in diameter and spaced apart not over 16 bar diameters, 48 tie diameters, or the least dimension of the column. When there are more than 4 vertical bars, additional ties should be provided so that every longitudinal bar will be held firmly in its designed position. See Fig. 77 (*a*).

The reinforcement for tied columns should be protected by a covering of concrete, cast monolithically with the core, for which the thickness should not be less than 1½″.

The maximum permissible axial load on *short* tied columns is given by the following formula:

$$P = 0.8 \times A_g(0.225f'_c + f_s p_g) \qquad (12)$$

in which P = the maximum permissible axial load, in pounds

A_g = the gross area of the column, in square inches

f'_c = the ultimate compressive strength of concrete, in pounds per square inch

f_s = the allowable compressive stress in vertical reinforcement, 16,000#/in^2 for intermediate grade steel and 20,000#/in^2 for rail or hard grade steel

p_g = the ratio of the effective cross-section area of vertical reinforcement to the gross column area A_g.

In practice the designer makes use of safe load tables * in designing columns. Without such tables it is necessary to take trial sections and compute the allowable loads they will support. Since the cross section of the vertical steel may vary from 0.01 to 0.04 of the gross column area and also because concrete of different strengths may be used, it is obvious that several different combinations may satisfy the requirements. To illustrate the use of the above formula let us compute the safe load on a specific column.

EXAMPLE

EXAMPLE. A reinforced concrete column has a 16″ x 16″ cross section and an unsupported length of 12′ 0″. The vertical reinforcement consists of eight ¾″ diameter rods of hard grade steel for which f_s = 20,000#/in^2; 2,000# concrete is used and f'_c = 2,000#/in^2. Let it be required to compute the allowable axial load the column will support.

SOLUTION. First let us see if this column comes under the classification of a *short column*. The dimension of the least side is 16″; 16 × 10 = 160″ = 13′ 4″. Since the column has a length of only 12′ 0″ the column is a short column and the above formula is applicable.

A ¾″ diameter rod has an area of 0.44in^2 (Table 31). Therefore 8 rods have an area of 8 × 0.44 or 3.52in^2. A_g, the gross area of the column, is 16 × 16 or 256in^2. Thus $p_g = \frac{3.52}{256} = 0.0137$. Note that this ratio falls between the limits of 0.01 and 0.04. Then

$$A_g = 256\text{in}^2$$

$$f'_c = 2{,}000\#/\text{in}^2$$

$$f_s = 20{,}000\#/\text{in}^2$$

$$p_g = 0.0137$$

$$P = 0.8 \times A_g(0.225f'_c + f_s p_g) \qquad (12)$$

* See Tables XX and XXI of the author's *Simplified Design of Reinforced Concrete.*

Substituting in this formula,

$$P = 0.8 \times 256\,[(0.225 \times 2{,}000) + (20{,}000 \times 0.0137)]$$

or

$$P = 147{,}913\#, \text{ the allowable axial load on the column.}$$

Lateral ties ¼″ in diameter, spaced 12″ on centers, meet the requirement for ties. The vertical rods must be placed not less than 1½″ from the face of the column.

PROBLEMS

102-A. Compute the allowable axial load and the tie spacing for a 14″ x 14″ tied column 11′ 6″ in length having eight ⅝″ diameter rods for the vertical reinforcement. $f'_c = 3{,}000\#/\text{in}^2$ and the reinforcement is of hard grade steel for which $f_s = 20{,}000\#/\text{in}^2$.

102-B. A short tied column 12″ x 12″ in cross section is reinforced with four ¾″ diameter rods of hard grade steel. If $f'_c = 2{,}000\#/\text{in}^2$, what allowable axial load will the column support? What should be the spacing of the ¼″ lateral ties?

103. **Spiral Columns.** A spiral column is a concrete column reinforced with vertical bars and closely spaced continuous spiral hooping, as indicated in Fig. 77 (*b*). When a concrete column is subjected to a load, there is a tendency for the concrete to expand in a direction perpendicular to the vertical axis of the column. To provide restraint against this tendency to burst, and also to support laterally the vertical reinforcing bars, spiral hooping is introduced. Although spiral reinforcement is limited almost entirely to columns of circular cross section, it may be employed in rectangular cross sections as well.

The ratio of the area of the vertical reinforcement to the gross column area p_g should not be less than 0.01 or more than 0.08. The minimum number of bars should be six and their minimum diameter ⅝″.

The maximum permissible axial load for spiral columns is found by the formula

$$P = A_g(0.225f'_c + f_s p_g) \qquad (13)$$

in which P = the maximum permissible axial load, in pounds
A_g = the gross area of the column, in square inches
f'_c = the ultimate compressive strength of concrete, in pounds per square inch
f_s = the allowable compressive unit stress in the vertical reinforcement, 16,000#/in^2 for intermediate grade steel and 20,000#/in^2 for rail or hard grade steel
p_g = the ratio of the effective cross-section area of the vertical reinforcement to the gross column area A_g.

The ratio of spiral reinforcement p' should not be less than the value given by formula (14), or less in any case than 0.0112 for hot-rolled spirals of intermediate grade or 0.0075 for cold-drawn wire spirals.

$$p' = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f'_s} \tag{14}$$

in which p' = the ratio of volume of spiral reinforcement to the volume of the concrete core (out to out of spirals)
A_g = the gross area of the column, in square inches
A_c = the area of the column core, in square inches
f'_c = the ultimate compressive unit stress of concrete
f'_s = useful limit stress of spiral reinforcement, to be taken as 40,000#/in^2 for hot-rolled rods of intermediate grade and 60,000#/in^2 for cold-drawn wire.

The spiral reinforcement should consist of evenly spaced continuous spirals held firmly in place by at least three vertical spacer bars. The center-to-center spacing of the spirals should not exceed one-sixth the core diameter and the clear spacing between spirals should not exceed 3″.

The column reinforcement should be protected everywhere by a covering of concrete cast monolithically with the core, for which the thickness must not be less than 1½″.

EXAMPLE

EXAMPLE. A short spiral column has an overall diameter of 17″ with 2″ of protecting covering for the vertical reinforcing rods. The vertical reinforcement consists of nine ¾″ diameter rods. Compute the allowable axial load the column will support and also the required spiral reinforcement. The spiral reinforcement will consist of cold-drawn wire for which $f'_s = 60{,}000\#/\text{in}^2$, $f'_c = 3{,}000\#/\text{in}^2$, and $f_s = 20{,}000\#/\text{in}^2$.

SOLUTION. The area of one ¾″ diameter rod is 0.44in^2 (Table 31). Therefore the area of nine rods is 9×0.44 or 3.96in^2. The gross diameter of the column is 17″; hence its gross area is $D^2 \times 0.7854$ or $A_g = 17 \times 17 \times 0.7854 = 226.98\text{in}^2$.

$$p_g = \frac{3.96}{226.98} = 0.01745$$

$$P = A_g(0.225f'_c + f_s p_g) \tag{13}$$

Substituting, $P = 226.98\,[(0.225 \times 3{,}000) + (20{,}000 \times 0.01745)] = 232{,}200\#$, the allowable axial load the column will support.

To determine the required spiral reinforcement we use the formula

$$p' = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f'_s} \tag{14}$$

As there will be a 2″ protecting covering, the diameter of the core is 17 − 4 or 13″. The area of the core A_c is $13 \times 13 \times 0.7854$ or 132.73in^2.

Substituting in the above formula,

$$p' \doteq 0.45\left(\frac{226.98}{132.73} - 1\right)\frac{3{,}000}{60{,}000} = 0.01597$$

the ratio of the volume of spiral reinforcement to the volume of the concrete core of the column.

As the area of the core is 132.73in^2, a 1″ vertical length of core contains 132.73in^3. A 1″ length of core requires a volume of spiral of 0.01597×132.73 or 2.12in^3.

Let us assume that the spiral will be made of ⅜″ diameter wire. The area of a ⅜″ diameter is 0.11in^2 (Table 31). The circumfer-

ence of the 13″ diameter core is 13 × 3.1416 or 40.84″. Thus the volume of one complete spiral is 40.84 × 0.11 or 4.5in³.

Now the vertical length of column to give 4.5in³ of ⅜″ spiral is $\frac{4.5}{2.12} = 2.1''$, the required pitch. Thus the spiral reinforcement will consist of ⅜″ cold-drawn wire with a 2″ pitch.

PROBLEM

103-A. An 18″ diameter column has vertical reinforcement consisting of ten ¾″ diameter rods. The vertical reinforcement has 2″ of protective covering. Compute the allowable axial load and the required spiral reinforcement. Specification data: $f'_c = 3{,}000\#/\text{in}^2$, $f_s = 20{,}000\#/\text{in}^2$, the spiral reinforcement to consist of ⅜″ cold-drawn wire for which $f'_s = 60{,}000\#/\text{in}^2$.

104. **Design of Concrete Stairs.** A concrete stairway may be considered as an inclined slab or beam. Since the stairs are usually built after the frame and floor slabs are in place, the stairway is

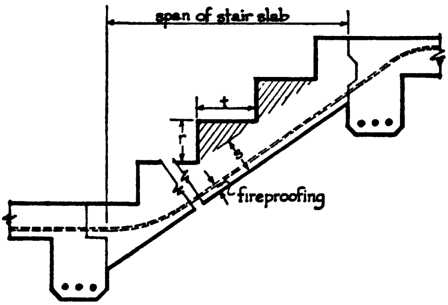


FIG. 78

figured as a simple beam having a maximum bending moment of $M = \frac{Wl}{8}$. Note particularly that the length of the span l is the *horizontal* distance between supports plus the approximate thickness of the slab. As in designing a floor slab with one-way reinforcement, a strip of stairway 12″ in width is considered and the depth and required steel area are computed in a similar manner. In addition to the tensile steel reinforcement, ⅜″ round rods are generally placed 18″ or 24″ on centers as cross reinforcement to prevent shrinkage and temperature cracks.

Many building codes require a live load for stairways of 100#/ft². Table 37 shows the total slab thickness and required reinforcement for various horizontal span lengths. This table was computed for 100#/ft² live load, and by its use proper depths and reinforcement may be selected without further computation.

Table 37. Concrete Stair Slabs for 100#/ft² Live Loads *

f_s = 20,000#/in² 1″ clear protective covering
f_c = 800#/in²
n = 15

Horizontal Span of Stairs in Feet	Total Thickness of Slab in Inches	Reinforcing Steel
4	3	⅜″φ 7½″ o.c.
5	3½	⅜″φ 6½″ o.c.
6	4	⅜″φ 5½″ o.c.
7	4½	⅜″φ 4½″ o.c.
8	5	½″φ 7½″ o.c.
9	5½	½″φ 6½″ o.c.
10	6	½″φ 5½″ o.c.
11	6½	½″φ 5 ″ o.c.
12	7	⅝″φ 7 ″ o.c.
13	7½	⅝″φ 6½″ o.c.
14	8	⅝″φ 6 ″ o.c.
5	9	⅝″φ 5½″ o.c.
16	9½	¾″φ 7 ″ o.c.
17	10	¾″φ 6½″ o.c.
18	10½	¾″φ 6 ″ o.c.

* Courtesy of the Portland Cement Association.

To compute the dead load, let r = height of a riser in inches, t = tread in inches, and s = the total thickness of the slab in inches, that is, the perpendicular distance from the underside of the slab to the foot of the riser. See Fig. 78. The weight of the steps (shaded area in figure), in pounds per horizontal square foot, figured at 144#/ft³, equals $(6 \times r)$ pounds.

$$\text{Weight of the slab} = s\sqrt{r^2 + t^2} \times \frac{12}{t} \text{ pounds}$$

EXAMPLE

EXAMPLE. A reinforced concrete stairway has twelve 7″ risers and eleven 11″ treads. Design the stairway for a live load of 100#/ft².
Specification data:

$$f_s = 20{,}000\#/\text{in}^2$$

$$f_c = 800\#/\text{in}^2$$

$$n = 15$$

SOLUTION. Assume the slab to be 6″ in thickness.

$$6 \times r = 6 \times 7 = 42\#/\text{ft}^2, \text{ the weight of the steps}$$

$$s\sqrt{r^2 + t^2} \times \frac{12}{t} = 6\sqrt{7^2 + 11^2} \times \frac{12}{11} = 85\#/\text{ft}^2, \text{ the weight of the slab}$$

If the live load = 100#/ft², the total load per horizontal square foot $w = 42 + 85 + 100 = 227\#$.

There are 11 treads at 11″ or 121″. Since the length of span is the horizontal distance between supports plus the thickness of the slab,

$$l = 10 + 0.5 = 10.5'$$

$$M = \frac{wl^2}{8} = \frac{227 \times 10.5 \times 10.5 \times 12}{8} = 37{,}540''\#$$

$$d = \sqrt{\frac{M}{Kb}} \qquad \text{(1) Art. 91}$$

$K = 131$ (Table 32) and $b = 12''$, the width of the strip under consideration.

$$d = \sqrt{\frac{37{,}540}{131 \times 12}} = 4.9, \text{ say } 5'', \text{ the effective depth of the slab}$$

Adding 1″ for fireproofing, 5 + 1 = 6″, the total thickness of the slab. See Fig. 78.

$$A_s = \frac{M}{f_s jd} \qquad (3) \quad \text{Art. 91}$$

$$A_s = \frac{37{,}540}{20{,}000 \times 0.875 \times 5} = 0.43\text{in}^2$$

the required area of steel reinforcement for each 12″ of width. Use ½″ round rods 5½″ on centers (Table 33).

$$u = \frac{V}{\Sigma_0 jd} \qquad (9) \quad \text{Art. 93}$$

$$u = \frac{1{,}135}{\left(\frac{12}{5.5} \times 1.57\right) \times 0.875 \times 5} = 76\#/\text{in}^2$$

Use plain bars with hooks at ends or continuous over slab supports. Note that this is the same design as obtained by use of Table 37.

In addition to the tensile reinforcement, place ⅜″ round rods for cross reinforcement in every other step.

PROBLEM

104-A. A reinforced concrete stair slab has thirteen 11″ treads and fourteen 6″ risers. Design the slab in accordance with the following specifications:

$$\text{live load} = 100\#/\text{in}^2$$

$$f_s = 20{,}000\#/\text{in}^2$$

$$f_c = 800\#/\text{in}^2$$

$$n = 15$$

105. **Design of Reinforced Concrete Footings.** DESIGN OF A WALL FOOTING. Reinforced concrete wall footings are designed by considering the projection beyond the face of the wall as a cantilever beam tending to bend upward. It is customary to compute the load on 1 linear foot of wall and to design the width, depth and required reinforcement for this portion of the footing. In this type of footing it is not desirable to use stirrups to resist diagonal tension, and the footing must therefore be designed accordingly.

The critical section for diagonal tension is at a distance d out from the face of the wall, d being the effective depth of the footing. As the bond stress in wall footings is high, the bond stress frequently determines the effective depth.

EXAMPLE

EXAMPLE. An 18″ wall exerts a load on a footing of 15,500 pounds per linear foot of wall. If the allowable soil load is 4,000#/ft², design a reinforced concrete footing in accordance with the following data:

f_s = 20,000#/in²

f_c = 800#/in²

v_c = limited to 60#/in² (special anchorage)

u = limited to 150#/in² (deformed bars anchored)

n = 15

SOLUTION. Assume the weight of the footing under 1 linear foot of wall to be 500#. Then 15,500 + 500 = 16,000#, the total load on the foundation bed per linear foot of wall.

Area of footing $= \frac{16{,}000}{4{,}000} = 4\text{ft}^2$. Since the strip of footing is 12″ wide its total length will be 4′.

Thickness of wall = 1.5′. Then $\frac{4 - 1.5}{2} = 1.25' = l$, the projection of the footing from the face of the wall. See Fig. 79 (a).

$M = \frac{wl^2}{2}$, the bending moment in the footing at the face of the wall, in which w = the upward pressure of the soil in pounds per square foot and l = the length of the projection of the footing in feet. Then

$$M = \frac{4{,}000 \times 1.25^2}{2} = 3{,}125'\# = 37{,}500''\#$$

$$d = \sqrt{\frac{M}{Kb}} \qquad \text{(1) Art. 91}$$

$K = 131$ (Table 32) and $b = 12''$, the width of the strip of footing. Then $d = \sqrt{\dfrac{37{,}500}{131 \times 12}} = 5''$. Accept $12''$ as the effective depth to avoid excessive bond stress.

The vertical shear used to determine the diagonal tension unit stress is taken at d distance from the face of the wall. As the length

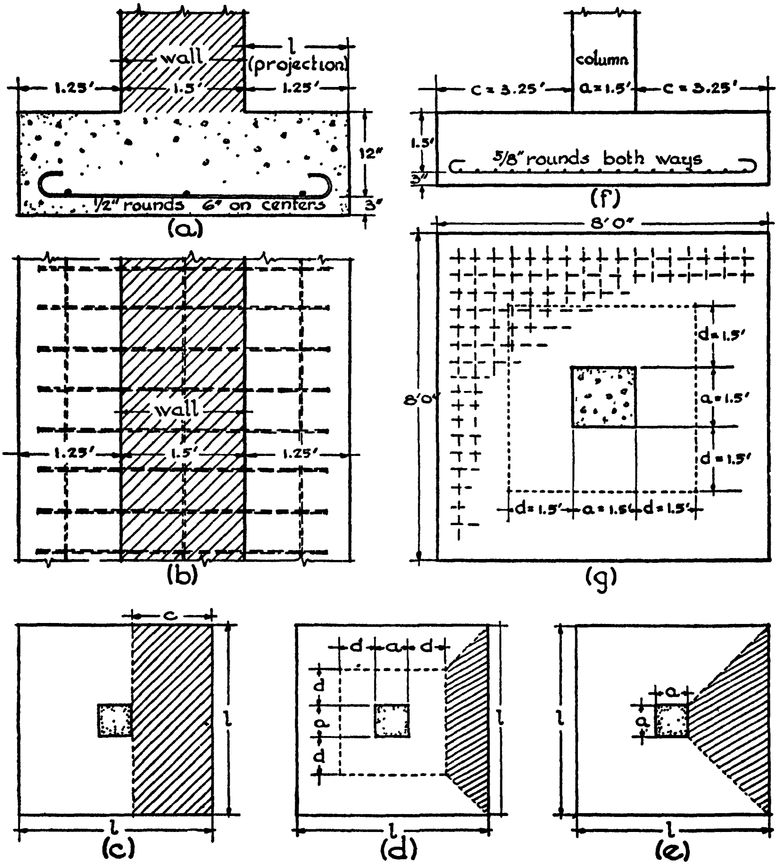


FIG. 79

of the projection is 1.25′, $1.25 - 1 = 0.25'$, the length of footing from the outer edge to the section to be tested for diagonal tension. Then $V = 4{,}000 \times 0.25 = 1{,}000\#$.

$$v = \frac{V}{jbd} = \frac{1{,}000}{0.875 \times 12 \times 12} = 7.9\#/\text{in}^2$$

This stress is well within the $60\#/\text{in}^2$ allowable.

In investigating the bond stress, V is taken at the face of the wall. Then $V = 1.25 \times 4{,}000 = 5{,}000\#$.

Let us assume that the tensile reinforcement will consist of ½″ round rods for which the perimeter is 1.57″ (Table 31). We shall let u, the bond stress, be $150\#/\text{in}^2$, its maximum value, and let x be the number of rods required for the 12″ strip of footing. Then

$$u = \frac{V}{\Sigma_0 jd} \quad \text{or} \quad 150 = \frac{5{,}000}{x \times 1.57 \times 0.875 \times 12}$$

$$x = \frac{5{,}000}{150 \times 1.57 \times 0.875 \times 12} \quad \text{and} \quad x = 2.01$$

the number of ½″ rods required for each 12″ length of footing. We shall accept, therefore, ½″ round rods 6″ on centers.

Now let us investigate the tensile stress in the ½″ rods with 6″ spacing. The area of a ½″ rod is 0.2in^2 (Table 31), and there are 2 rods in each 12″ strip of footing. Then

$$A_s = \frac{M}{f_s jd} \quad \text{or} \quad f_s = \frac{M}{A_s jd}$$

or

$$f_s = \frac{37{,}500}{2 \times 0.2 \times 0.875 \times 12} = 8{,}930\#/\text{in}^2$$

the actual tensile unit stress. Note that this stress is well within the $20{,}000\#/\text{in}^2$ allowable and that the effective depth has been determined by the bond stress, which will be its maximum allowable value.

For a protection to the reinforcement, 3″ of concrete will be placed below the rods, making a total depth of 12 + 3 or 1′ 3″. Three ⅜″ round shrinkage and temperature rods will be placed in the footings parallel to the wall. See Fig. 79 (*a*).

Design of an Independent Column Footing. The great majority of independent column footings are square in plan, having two sets of reinforcing bars at right angles with each other. This

is known as two-way reinforcement. The column may be placed directly on the footing block or it may be supported by a pedestal.

The area of the footing is found by dividing the column load plus the approximated weight of the footing by the allowable bearing capacity of the foundation bed. The weight of independent column footings varies from 6 to 11 per cent of the column load.

The stresses for shear and bond are generally the critical stresses in the design of such footings. The customary procedure is to compute the effective depth by the formula $d = \sqrt{\frac{M}{Kb}}$ and then to increase d arbitrarily so that the bond and shear stresses come within their prescribed limits.

There is a great discrepancy in code requirements in connection with the design of column footings. The example that follows employs the methods of procedure required by many codes and may be used with safety.

The maximum bending moment is the moment that results from the resisting upward pressure of the foundation bed at the face of the column (or pedestal) on the hatched area shown in Fig. 79 (c). The area is ($c \times l$) square feet, and if w is the upward pressure in pounds per square foot, the total upward force is ($c \times l \times w$). The lever arm of the moment about the face of the column is $\frac{c}{2}$; hence the moment is $\frac{wlc^2}{2}$ foot-pounds.

The critical section for shear as a measure of diagonal tension is taken at a vertical section d distance from the face of the column (or pedestal). In Fig. 79 (d), $V = \frac{l^2 - (a + 2d)^2}{4} \times w.$

In computing the bond stress, the footing area used to determine V is the hatched area indicated in Fig. 79 (c). Thus

$$V = \frac{l^2 - a^2}{4} \times w.$$

Reinforcement for two-way footings consists of deformed bars with hooked ends. The outer faces of the hooks should not be less than 3″ or more than 6″ from the face of the footing. A distance of 3″ below the reinforcing rods is considered to be a minimum thickness of protection.

EXAMPLE

EXAMPLE. An 18″ square column is supported directly by the footing block. The load on the column is 240,000# and the allowable bearing capacity of the foundation bed is 4,000#/ft². Let it be required to design a square two-way block footing in accordance with the following specification data:

f_s = 20,000#/in²

f_c = 900#/in²

v_c = limited to 60#/in²

u = limited to 112#/in² for deformed bars with hooked ends

SOLUTION. Assume the footing to weigh 16,000#. Then 240,000 + 16,000 = 256,000#, the total load on the foundation bed. As the allowable bearing capacity of the foundation bed is 4,000#/ft², 256,000 ÷ 4,000 = 64ft², the required area of the footing. We shall accept a footing 8′ x 8′ in plan. See Fig. 79 (*g*).

The weight of the footing does not contribute to its tendency to bend; hence the net upward pressure w is 240,000 ÷ 64 or w = 3,750#/ft².

$$M = \frac{wlc^2}{2} = \frac{3{,}750 \times 8 \times 3.25 \times 3.25}{2} = 159{,}000'\# = 1{,}908{,}000''\#$$

In accordance with the specification data, K = 157 (Table 32). Then $d = \sqrt{\frac{M}{Kb}}$ or $d = \sqrt{\frac{1{,}908{,}000}{157 \times 96}}$ and d = 11.4″. We shall accept temporarily an effective depth of 18″; see Fig. 79 (*f*). The shear and bond stresses to be investigated later will show whether or not the assumed depth is sufficiently great.

V to be used in computing the shearing unit stress is found by the formula

$$V = \frac{l^2 - (a + 2d)^2}{4} \times w = \frac{8^2 - (1.5 + 3)^2}{4} \times 3{,}750 = 41{,}000\#$$

$$v = \frac{V}{jbd} = \frac{41{,}000}{0.875 \times 54 \times 18} = 49\#/\text{in}^2$$

In the above formula 54 is 4.5′ expressed in inches.

As 49#/in^2 is less than 60#/in^2, the allowable, the shearing unit stress is acceptable.

In two-way footings, when the maximum bending moment has been computed as previously described, the customary formula is used to determine the area of the tensile reinforcement but only 85 per cent of the bending moment is taken. Thus

$$A_s = \frac{M}{f_s jd} = \frac{0.85 \times 1{,}908{,}000}{20{,}000 \times 0.866 \times 18} = 5.18\text{in}^2$$

The cross-section area of a ⅝″ round rod is 0.31in^2 (Table 31). Then 5.18 ÷ 0.31 = 16.7; accept seventeen ⅝″ round rods.

V used to determine the bond stress is

$$V = \frac{l^2 - a^2}{4} \times w = \frac{8^2 - 1.5^2}{4} \times 3{,}750 = 57{,}750\#$$

The perimeter of a ⅝″ rod is 1.96″ (Table 31). Then

$$u = \frac{V}{\Sigma_0 jd} = \frac{57{,}750}{17 \times 1.96 \times 0.875 \times 18} = 110\#/\text{in}^2,$$

the unit bond stress

This stress is acceptable since it is less than 112#/in^2, the allowable.

The reinforcement will consist of two bands of seventeen ⅝″ round deformed bars at right angles to each other; the ends will have standard hooks. The effective depth of the footing will be 18″, and 3″ of concrete will be placed below the bottom layer of rods for a protective covering.

PROBLEMS

105-A. A masonry wall 2′ 0″ in thickness exerts a load of 34,400 pounds per linear foot of wall on the foundation bed for which the bearing capacity is 6,000#/ft^2. Design a reinforced concrete footing for the wall in accordance with the following specification data:

f_s = 20,000#/in^2

f_c = 800#/in^2

v_c = limited to 60#/in^2

u = limited to 150#/in^2

n = 15

105-B. A 15″ square column supports a load of 138,000# including its own weight. If the bearing capacity of the foundation bed is 3,000#/ft², design a square independent footing in accordance with the following specification data:

f_s = 20,000#/in²

f_c = 900#/in²

v_c = limited to 60#/in²

u = limited to 112#/in²

n = 15

SECTION V

ROOF TRUSSES

106. **Roof Trusses.** A truss is a framed structure composed generally of straight members so arranged and fastened together at their ends that the stresses in the members, due to loads at the joints, are either tension or compression. Most of the trusses in practice are composed of a number of triangles framed together.

Definition of terms used in connection with trusses follows.

A *bay* is that portion of a roof which occurs between two adjacent trusses, the spacing of the trusses on centers being the width of a bay. A *purlin* is the beam, spanning from truss to truss, which transfers the loads due to roofing material, snow, and so on, to the truss. The portion of a truss which occurs between two adjacent joints of the upper chord is called a *panel.* The load occurring at a joint or panel point is, therefore, the length of a bay in feet multiplied by the width of a panel in feet multiplied by the roof load in pounds per square foot. The height divided by the span is called the *pitch* of the roof. Other names given to a typical roof truss are as noted in Fig. 80 (*a*).

107. **Force Polygon.** In designing a roof truss the designer must first determine the magnitude and character of the stresses in the members. By character is meant the kind of stress, tension, or compression. This may be accomplished readily by graphical methods.

With the exception of wind loads, trusses are generally symmetrically loaded. In the truss shown in Fig. 80 (*a*), for example, there would be three equal vertical panel loads, W pounds each; and, since the truss is symmetrical, each upward force or reaction would equal $\frac{1}{2} \times 3W = 1\frac{1}{2}W$. The lower left-hand end or heel of the truss may be represented diagrammatically as in Fig. 80 (*b*). The forces, read clockwise about the joint, are three in number: AB, the reaction an upward force of $1\frac{1}{2}W$ pounds; BC,

the upper chord, and *CA*, the lower chord, both of which are of unknown magnitude. These forces are concurrent, and since by data they are in equilibrium, a stress diagram corresponding to the forces must close. Therefore draw an upward force *ab* to some convenient scale representing the reaction $1\frac{1}{2}W$, Fig. 80 (*c*). From point *b* draw a line parallel to *BC*, and from point *a* draw a line parallel to *CA*. The intersection of these two lines determines the point *c*. The magnitude of the stresses in the members *BC* and *CA*

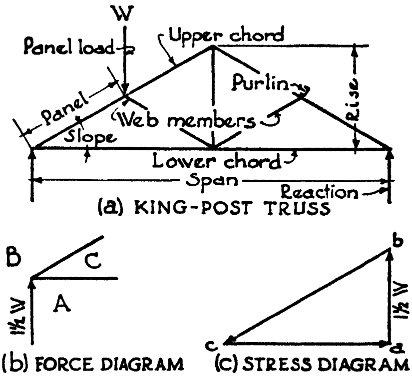


Fig. 80

is found by scaling their lengths in the stress diagram, Fig. 80 (*c*), at the same scale at which *ab* was drawn.

To determine the character of the stresses at this joint we refer to both Figs. 80 (*b*) and 80 (*c*). Consider first the member *BC* about the point *ABC*. In the stress diagram we find that *bc* reads downward to the left. In the force diagram, if *BC* reads downward to the left this is *toward* the reference point *ABC* and hence the member *BC* is in compression. Next consider the member *CA* about the point *ABC*. In the stress diagram we find that *ca* reads from left to right, and if we consider the member *CA* in the force diagram as reading from left to right we read *away* from the reference point *ABC* and therefore the member *CA* is in tension.

The length of a member in a truss bears no direct relation to the magnitude of its stress, the magnitude of the stress being determined by the length of the line in the stress diagram corresponding to the truss member.

The stress diagram, Fig. 80 (*c*), is the force polygon for the three concurrent forces at the heel of the truss. The stress diagram for the entire truss would consist of combined force polygons of the forces at all the different joints of the truss. Figure 81 (*a*) is a

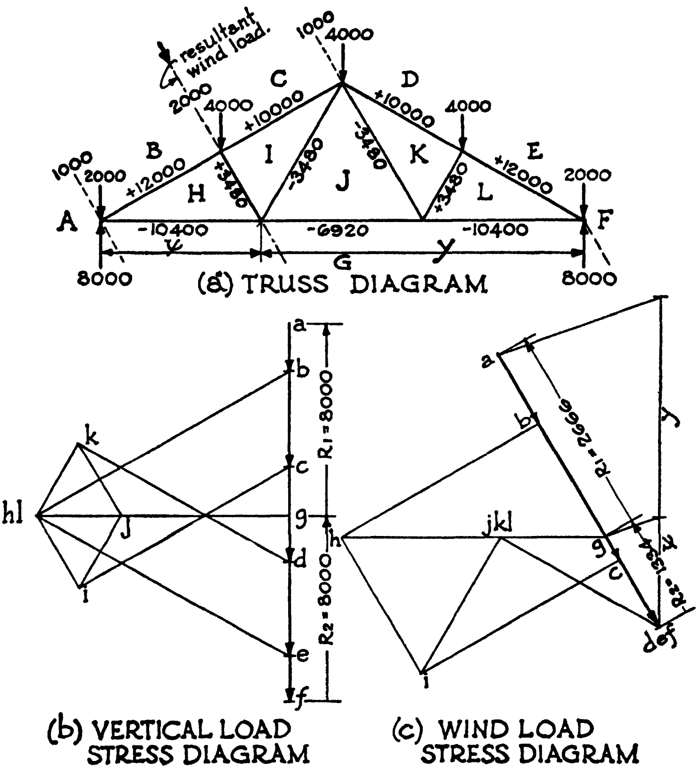


FIG. 81

four-panel Fink truss, and the complete stress diagram for vertical loads is shown in Fig. 81 (*b*).

108. Stress Diagrams. The panel load for the vertical loading in the truss shown in Fig. 81 (*a*) is 4,000#. The two end loads are 2,000# each, making a total vertical load of 4,000 + 4,000 + 4,000 + 2,000 + 2,000 or 16,000#. Since the truss is symmetrical, each reaction or upward force at the support is $\frac{16,000}{2} = 8,000\#$.

The panel loads and reactions being known, *the first step in drawing a stress diagram is to draw a force polygon of the external forces.* These forces are *AB*, *BC*, *CD*, *DE*, *EF*, *FG*, and *GA*, and the magnitudes of all are known. Therefore, at a suitable scale, draw *ab*, Fig. 81 (*b*), a downward force equal to 2,000#. The next external force is *BC*, and from the point *b* just determined draw *bc* downward equivalent to 4,000#. Continue similarly with *CD*, *DE*, and *EF*. This completes the downward forces. The line just drawn is called the load line, the length being equivalent to 16,000#. The next external force is *FG*, an upward force of 8,000#. This determines the position of the point *g*, and *GA*, the remaining external force, completes the polygon of the external forces. Since the loads and reactions are vertical the force polygon is a vertical line.

In conjunction with the polygon just drawn, draw a force polygon for the forces *AB*, *BH*, *HG*, and *GA* about the point *ABHG*. From *b* draw a line parallel to *BH*, and from *g* draw a line parallel to *HG*. Their intersection determines the point *h*. Next consider the members about the joint *BCIH*. From *c* draw a line parallel to *CI*, and through *h* draw a line parallel to *IH*, their intersection determining point *i*. The next joint is *HIJG*. Through *i* draw a line parallel to *IJ*, and through *g* draw a line parallel to *JG*, thus establishing the point *j*. In the same manner take the joints *CDKJI* and *DELK*. This completes the stress diagram, Fig. 81 (*b*). The magnitudes of the stresses in the members are found by scaling the lengths of the lines in the stress diagram just completed, and the character of the stresses is found as explained in the preceding article. A member in compression is indicated by the plus sign (+) and tension members by minus (−). For the vertical loads the character and magnitude of the stresses are shown on the truss diagram, Fig. 81 (*a*).

109. Wind-Load Stress Diagram. The stresses in members of a roof truss due to wind loads are found by constructing a stress diagram as previously described. It is assumed that the wind exerts a pressure perpendicular to the roof surface. In the truss diagram, Fig. 81 (*a*), the wind is shown coming from the left, indicated by dotted lines, the total load being 1,000 + 2,000 + 1,000 or 4,000#. To draw a stress diagram for the wind loads proceed as before. Construct the force polygon of the external forces, namely *AB*, *BC*, *C–DEF*, *DEF–G*, and *GA*, the latter two

forces being the wind-load reactions. It will be noted that since the wind comes from the left there are no forces *DE* and *EF*; consequently the letters *D*, *E*, and *F* represent a single point in the stress diagram. Draw *ab*, *bc*, and *c–def*, Fig. 81 (*c*). It may be assumed that the reactions due to wind loads are parallel to the direction of the wind and, since the wind comes from the left, the left-hand reaction will have a greater magnitude than that on the right for this particular truss. For the purpose of finding the magnitudes of the reactions we may consider that all the wind loads are concentrated in a single line of action, at *BC*. This resultant wind load continued divides the lower chord into two parts x and y, and the magnitudes of the reactions are to each other as the division x is to y. The line *a–def*, representing the total wind load, is therefore to be divided in the same proportion as the two divisions of the lower chord, x and y. To accomplish this, from the point *def* erect a line whose length is equivalent to the length of the lower chord and divide it into sections x and y. From the extremity of the line just drawn, draw a line to point a and a parallel line from the point separating x and y to the load line. This determines the point g and consequently the reactions R_1 and R_2. The force polygon of the external forces is now completed, and the stress diagram is then drawn as previously described. It will be found that the letters j, k, and l fall at one point, indicating that there are no stresses in *JK* and *KL* because the wind comes from the left.

In designing the various members of the truss the design loads are found by adding together the vertical and wind-load stresses. For instance, the stress in the member *HI* due to the vertical load is 3,480#, and 2,000# is the stress due to the wind load. The design load in *HI* will be, therefore, 3,480 + 2,000 or 5,480#. Although there is no stress in *KL* as the wind comes from the left, the section used for *HI* will be used for *KL* since the direction of the wind may be reversed.

110. Roof Loads. The dead load is always present on roof trusses. It consists of the roof covering such as slate, felt, sheathing, rafters, purlins, and the weight of the truss. Table 38 gives approximate weights of some of the materials commonly used. The actual weight of the truss cannot be determined accurately in

advance but it may be approximated by means of Table 39. The approximate weight of the truss having been computed, it is distributed among the panel loads on the upper chord. After the truss has been designed its actual weight may be computed and compared with the assumed weight. If the two are not in reasonable agreement another weight should be assumed and the truss redesigned. Except for unusual cases this step is seldom necessary.

Table 38. Approximate Weights of Roofing Material

Roofing Material	Approximate Weight per Square Foot, Pounds
Corrugated galvanized iron	
20 gage	2.0
18 gage	3.0
Nailing concrete, per inch of thickness	8.0
Sheathing	
White pine, spruce and hemlock, 1″ thick	2.5
Southern yellow pine, 1″ thick	3.5
Shingles	
Wood	3.0
Asbestos	5.0
Slag roof, 4-ply felt	5.0
Slate	
3/16″ thick	7.0
3/8″ thick	12–14
Tile, clay	
Plain	15.0
Spanish	10.0
Tin plate, IX thickness	0.75

Since the snow load is vertical, it is generally added to the dead load for the purpose of drawing the vertical load stress diagram. The magnitude of the snow load depends on the pitch of the roof and the locality. Table 40 gives snow loads for various pitches and locations and may be used in safety when no specific loads are given by local building codes. For accuracy, a stress diagram may be drawn for snow loads alone, the stresses in the truss members being directly proportional to those for dead loads.

Table 39. Approximate Weight of Steel Trusses in Pounds per Square Foot of Horizontal Projection

Span	Pitch of Roof			
Feet	45°	30°	25°	Flat
Up to 40	5	6	7	8
40–50	6	7	7	8
50–60	7	8	9	10
60–70	7	8	9	10
70–80	8	9	10	11

Table 40. Snow Loads for Roof Trusses in Pounds per Square Foot of Roof Surface

Locality	Pitch of Roof				
	45°	30°	25°	20°	Flat
Northwestern and New England States....	15	20	30	35	40
Western and Central States..............	10	15	25	30	35
Pacific and Southern States.......... ...	0	5	10	10	10

The wind pressure, when coming at right angles to the side of a building, probably never exceeds 40#ft^2. Since it is customary to consider that the wind acts in a direction perpendicular to the roof surface, the magnitude of the wind pressure decreases as the pitch becomes smaller. Table 41 gives wind pressures for roofs of various pitches based on a wind pressure of 40#/ft^2 on a vertical surface.

Obviously a condition of dead load, maximum snow, and maximum wind load cannot occur simultaneously. The conditions

Table 41. Wind Pressure on Roof Surfaces

Pitch of Roof	Normal Pressure
Degrees	Pounds per square foot
15	14.0
20	18.0
25	22.0
30	26.0
35	30.0
40	33.0
45	36.0
50	38.0

causing maximum stresses most likely to occur are the following combinations:

(1) Dead load and maximum snow.
(2) Dead load, maximum wind, and minimum snow.
(3) Dead load, minimum wind, and maximum snow.

For accuracy, one may draw separate stress diagrams for dead, snow, and wind loads, tabulate the stresses, and use for the design load for the various truss members the greatest stress which is found for any of the above combinations. It is customary to consider ½ the maximum snow and wind loads as the minimum values.

111. **Equivalent Vertical Loads. Design of a Fink Truss.** A convenient and practical method of computing stresses in truss members is to use an equivalent vertical load for snow and wind loads. This load may be added to the dead load, and hence only one stress diagram need be drawn. For the usual simple roof trusses resting on masonry walls the results obtained by this method are quite similar to those obtained by combined dead, snow, and wind loads previously described, and a considerable saving of time is effected. Equivalent vertical loads are given in Table 42. In using this method the selection of the proper equivalent load depends on the judgment and experience of the designer, but the values given in Table 42 will prove to be adequate for the usual conditions met in practice.

Table 42. Equivalent Vertical Loads for Combined Snow and Wind Loads in Pounds per Square Foot of Roof Surface

Locality	Pitch of Roof				
	45°	30°	25°	20°	Flat
Northwestern and New England States..	28	25	24	35	40
Western and Central States.... . . .	28	25	24	30	35
Southern and Pacific States....	28	25	24	22	20

EXAMPLE

EXAMPLE. Design a steel 8-panel Fink truss having a clear span of 60′ with trusses spaced 18′ on centers. The roof construction consists of ⅜″ slate laid on a 4″ slab of nailing concrete. The building is to be located in one of the Central States. See Fig. 82 (*a*).

SOLUTION. The number of square feet of roof surface for each panel point is 8.6′ x 18′ or 154.8ft^2.

An equivalent vertical load of 25#/ft^2 for combined snow and wind loads being used (Table 42), the panel loads are computed thus:

⅜″ slate roofing	= 12	(Table 38)
4″ nailing concrete	= 32	(Table 38)
Snow and wind equivalent	= 25	(Table 42)
Total	= 69#/ft^2	

The vertical load at each panel, not including the weight of the truss, will be, therefore, 154.8 × 69 or 10,681#.

The horizontal projection of the roof area supported by one truss is 60′ x 18′ or 1,080ft^2. Since the approximate weight of the truss is 8#/ft^2 (Table 39), the estimated weight is 1,080 × 8 = 8,640#.

If the weight of the truss is assumed to be distributed among the panel points, 8,640 ÷ 8 = 1,080#, the estimated weight of the truss per panel point. Then the vertical load at each panel point will be 10,681 + 1,080 or 11,761#. The total load on the truss will be

8 × 11,761 or 94,088#, and each reaction or load on the support will be ½ × 94,088 or 47,044#.

As the truss is symmetrically loaded, it will be necessary to draw only half of the stress diagram. First the force polygon of the external forces is drawn as explained in Art. 108; see Fig. 82 (*b*).

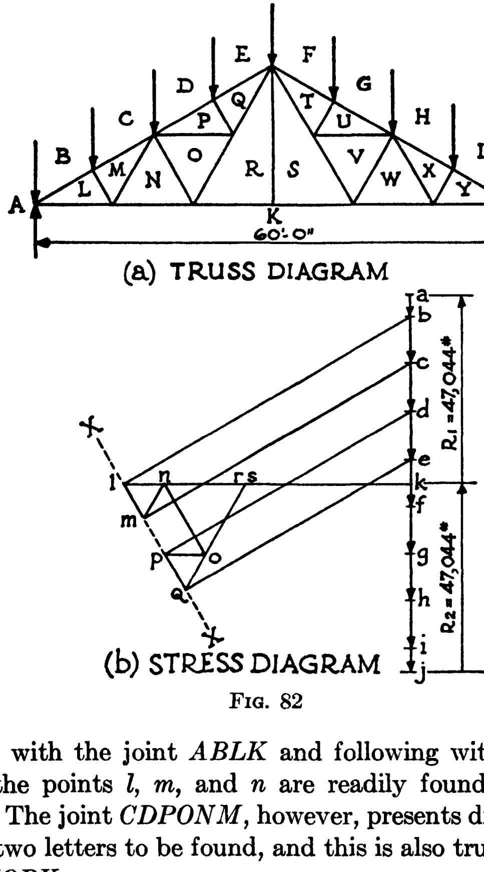


FIG. 82

Beginning with the joint *ABLK* and following with *BCML* and *LMNK*, the points *l*, *m*, and *n* are readily found in the stress diagram. The joint *CDPONM*, however, presents difficulties since there are two letters to be found, and this is also true with respect to joint *NORK*.

The Fink truss of this type requires a special solution, one of the simplest methods being as follows. It is known that each successive member of the upper chord, beginning with the one nearest the support, has the same rate of decrease in stress. In this instance the stresses in *BL* and *CM* have been determined, and by referring

to the stress diagram the decrease in stress is observed. The stresses in *DP* and *EQ* will be reduced in the same ratio, and their magnitudes are found by drawing the dotted line *X–X* through points *l* and *m* in the stress diagram. The intersection of a line drawn through *d*, parallel to *DP*, and the line *X–X* determines the point *p*; and a line through *e* parallel to *EQ* intersects *X–X* at *q*. Determining the points *n*, *o*, and *r* presents no difficulties. This completes one-half the stress diagram and is sufficient for our purpose since the truss is symmetrically loaded and the stresses in the members in each half of the truss are alike. It should be noted that no stress exists in the member *RS* but a small steel angle will be used to prevent a possible deflection of this portion of the lower chord.

Table 43. Stresses in Fink Truss

Truss Member	Character of Stress	Magnitude of Stress	Section
		pounds	
BL	+	81,900	2 Ls 4″ x $3\frac{1}{2}$″ x $\frac{7}{16}$″
CM	+	76,050	"
DP	+	70,200	"
EQ	+	64,350	"
KL	−	71,010	2 Ls $3\frac{1}{2}$″ x $2\frac{1}{2}$″ x $\frac{7}{16}$″
KN	−	60,840	"
KR	−	40,480	2 Ls 3″ x $2\frac{1}{2}$″ x $\frac{5}{16}$″
LM	+	10,170	2 Ls $2\frac{1}{2}$″ x 2″ x $\frac{5}{16}$″
PQ	+	10,170	"
NO	+	20,240	2 Ls $3\frac{1}{2}$″ x $2\frac{1}{2}$″ x $\frac{5}{16}$″
MN	−	10,170	2 Ls $2\frac{1}{2}$″ x 2″ x $\frac{5}{16}$″
OP	−	10,170	"
OR	−	20,240	"
QR	−	30,420	"
RS	0	0	1 L $2\frac{1}{2}$″ x 2″ x $\frac{5}{16}$″

Table 43 is a tabulation of the stresses found in the Fink truss. The first column designates the member, the second the character of the stress, and the third the magnitude of the stress in the various members. These stresses are found by measuring the lengths of the lines in the stress diagram, the same scale used in laying off the force polygon of external forces being used.

112. **Compression Members.** Steel trusses of this size generally have members made up of two angles separated at the connections by $\frac{3}{8}''$ or $\frac{1}{2}''$ gusset plates. Generally the most efficient section consists of two angles having unequal legs, with the long legs placed back to back. The compression members are struts and are designed as columns of given length and axial loads. The designer may use whatever column formula is required by the building code, as described in Art. 78. Tables 27, 28, and 29 give allowable axial loads for sections formed of two angles with a $\frac{3}{8}''$ space between backs. These tables are computed in accordance with the column formula given in Art. 79, and the proper section may be selected by referring to the tables without further computations. It should be remembered that the limiting value of $\frac{l}{r}$ for main compression members is 120, and frequently the size of a column is determined by this ratio rather than by the magnitude of the load. In the tables giving allowable loads for columns, the loads to the right of the heavy vertical lines are for columns whose slenderness ratio is greater than 120. For instance, a strut made up of two 4″ x 3″ x $\frac{5}{16}''$ angles, long legs back to back and $\frac{3}{8}''$ apart, has a least radius of gyration of 1.27″, and the slenderness ratio of a column 14′ long is $\frac{l}{r} = \frac{14 \times 12}{1.27} = 132$. If the length is 12′, $\frac{l}{r} = \frac{12 \times 12}{1.27} = 113$. Refer to Table 29, and note that the heavy vertical line occurs between these two lengths.

If the length of the upper chord is not too great it is customary to use the same cross section for the entire length. The member *BL* has a compressive stress of 81,900#. The remaining members of the upper chord have smaller stresses so that the stress in *BL* will determine the size of the entire upper chord.

EXAMPLE

Example. Determine the size of the compression members in the Fink truss shown in Fig. 82, using $\frac{3}{8}''$ gusset plates.

Solution. The stress in *BL* is 81,900# (see Table 43), and its length is 8′ 4″. The method of designing steel columns as explained in Art. 79 will be followed and the column formula used in compiling Tables 27, 28, and 29 will be employed.

STEP 1. Assume two 4″ x 3½″ x 7/16″ angles, long legs back to back and ⅜″ apart as a trial section. From Table 29, the least $r = 1.24''$, and the area of the cross section $= 6.18\text{in}^2$.

STEP 2. $\frac{l}{r} = \frac{(8 \times 12) + 4}{1.24} = 80.$

STEP 3.

$$f = \frac{18{,}000}{1 + \frac{1}{18{,}000}\left(\frac{l}{r}\right)^2} = \frac{18{,}000}{1 + \frac{1}{18{,}000}(80)^2} = 13{,}279\#/\text{in}^2$$

STEP 4. $13{,}279 \times 6.18 = 82{,}064\#$.

STEP 5. Since the design load is 81,900# and the allowable load on the assumed section is 82,064#, the section is acceptable.

By referring to Table 29 we could have selected the proper section without computations. We find that, for the section assumed, a column 8′ in length will support safely 83,000#. Accept two 4″ x 3½″ x 7/16″ angles, long legs back to back. This section will be used for the entire length of the upper chord.

The member *ON* has a length of 10′ and a load of 20,240# (Table 43). Referring to Table 29 select two 3½″ x 2½″ x 5/16″ angles. Note that this section will support 39,000# and that $\frac{l}{r}$, the slenderness ratio, is the governing factor.

The design loads for members *LM* and *PQ* is 10,170# and their length is 5′. The minimum section used in the truss will be two 2½″ x 2″ x 5/16″ angles, and by referring to Table 29 we find it acceptable for these two members.

113. Members in Tension. If a member in tension fails, the rupture will occur at the section where the area is smallest. This least cross section will occur at a rivet hole. In trusses of this type, ¾″ and ⅞″ rivets are generally used. In computing the net area, the rivet hole is considered to be ⅛″ greater than the nominal size of the rivet. It should be noted that the length of a member in tension is not a factor in determining its cross section. Table 44 gives the effective net areas for *single* angles with one rivet hole deducted. Since two angles are generally used for truss members the proper section is readily found by dividing one-half the design

Table 44. Effective Net Areas of Angles with One Rivet Hole Deducted

Size, In	Gross Area, In^2	Net Area, In^2		Size, In	Gross Area, In^2	Net Area, In^2	
		7/8" rivets	3/4" rivets			7/8" rivets	3/4" rivets
2½ x 2 x 3/16	0.81		0.64	5 x 3 x 5/16	2.40	2.09	2.13
1/4	1.06		0.84	3/8	2.86	2.48	2.53
5/16	1.31		1.04	7/16	3.31	2.87	2.92
				1/2	3.75	3.25	3.31
3 x 2½ x 1/4	1.31	1.06	1.09	9/16	4.18	3.62	3.69
5/16	1.62	1.31	1.35	5/8	4.61	3.98	4.06
3/8	1.92	1.54	1.59	11/16	5.03	4.34	4.43
7/16	2.21	1.77	1.82	3/4	5.44	4.69	4.78
3½ x 2½ x 1/4	1.44	1.19	1.22	5 x 3½ x 5/16	2.56	2.25	2.29
5/16	1.78	1.47	1.51	3/8	3.05	2.67	2.72
3/8	2.11	1.73	1.78	7/16	3.53	3.09	3.14
7/16	2.43	1.99	2.04	1/2	4.00	3.50	3.56
1/2	2.75	2.25	2.31	9/16	4.47	3.91	3.98
				5/8	4.92	4.29	4.37
4 x 3 x 5/16	2.09	1.78	1.82	11/16	5.37	4.68	4.77
3/8	2.48	2.10	2.15	3/4	5.81	5.06	5.15
7/16	2.87	2.43	2.48				
1/2	3.25	2.75	2.81	6 x 3½ x 3/8	3.42	3.04	3.09
9/16	3.62	3.08	3.13	7/16	3.97	3.53	3.58
5/8	3.98	3.35	3.43	1/2	4.50	4.00	4.06
				9/16	5.03	4.47	4.54
4 x 3½ x 5/16	2.25	1.94	1.98	5/8	5.55	4.92	5.00
3/8	2.67	2.29	2.34	11/16	6.06	5.37	5.46
7/16	3.09	2.65	2.70	3/4	6.56	5.81	5.90
1/2	3.50	3.00	3.06	13/16	7.06	6.25	6.35
5/8	4.30	3.67	3.75	7/8	7.55	6.67	6.78
11/16	4.68	3.99	4.08				
3/4	5.06	4.31	4.40				

load by the allowable tensile stress and selecting the required section directly from the table.

EXAMPLE

Example. Design the tension members in the Fink truss shown in Fig. 82, using ¾" rivets and an allowable unit tensile stress of 18,000#/in^2.

Solution. The member *LK* has a design load of 71,010#; its length is not a factor in its design. Assume the member to be composed of two 3½" x 2½" x ⅜" angles. The gross area of one angle is 2.11in^2 (Table 44). A ¾" rivet requires that the diameter of the hole to be considered is ¾ + ⅛ or ⅞". The thickness of the angle is ⅜", and therefore the area to be deducted from the gross area of

the angle is $\frac{7}{8} \times \frac{3}{8}$ or 0.328in^2. The area that will resist tension is therefore the gross area minus the rivet hole or 2.11 − 0.328 or 1.782in^2. If the allowable tensile unit stress on the section is 18,000#/in^2 the allowable net stress on the section will be 1.782 × 18,000 or 32,076#. There are, however, two angles, and the allowable tensile stress on the entire cross section will be 2 × 32,076 or 64,152#. Since the design load is 71,010# the assumed section is too small and a larger section is required. By using the same method it is found that two 3½″ x 2½″ x $\frac{7}{16}$″ angles have an allowable net stress of 73,600#. This section is acceptable.

The method just explained necessitates taking trial sections. Note that the proper sections may be selected directly by referring to Table 44. In this instance the tensile load on one angle is 71,010 ÷ 2 or 35,505#. As the allowable tensile stress is 18,000#/in^2, 35,505 ÷ 18,000 = 1.97in^2, the required net area. Table 44 shows that a 3½″ x 2½″ x $\frac{7}{16}$″ angle punched for a ¾″ rivet has a net area of 2.04in^2; hence two 3½″ x 2½″ x $\frac{7}{16}$″ angles are accepted.

The member *KN* has a design load of only 60,840# but the same section will be used, namely, two 3½″ x 2½″ x $\frac{7}{16}$″ angles, the member being continuous up to the joint *NORK*.

The design load on member *KR* is 40,480#. One half of 40,480 is 20,240#, the load to be resisted by one angle. 20,240 ÷ 18,000 = 1.12in^2, the required net area. Table 44 shows that a 3″ x 2½″ x $\frac{5}{16}$″ angle has a net area of 1.35in^2 and therefore two 3″ x 2½″ x $\frac{5}{16}$″ angles are accepted.

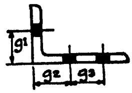
The remaining members in tension have relatively small stresses, and since sections composed of two 2½″ x 2″ x $\frac{5}{16}$″ angles will be the minimum section used in the truss, this section will be used for members *MN*, *OP*, *OR*, and *QR*.

PROBLEM

113-A. An eight-panel Fink truss has a span of 70′ 0″ and a pitch of 30°. The total vertical load on the truss is 104,000#; thus panel loads are 13,000# each, except those over the supports which are each 6,500#. Using the lettering shown in Fig. 82 (*a*), tabulate the character and magnitude of the stresses in the various members and also the structural steel sections to be used. For struts, use Table 29. Assume that the allowable tensile stress of structural steel is 20,000#/in^2 and that ¾″ rivets are to be used.

114. **Riveted Joints.** The design of riveted connections is discussed in Art. 66 and the number of rivets to be used at various joints is determined by the method therein described. In general the required number of rivets at the end of a member is the design load on the member divided by the controlling value of one rivet. To determine the number of rivets one must know the design load, thickness of the gusset plate and angles, the size of the rivets, and the allowable unit stresses.

Table 45. Gage Dimensions for Standard Angles



Leg	2	2½	3	3½	4	5	6	7	8
g_1..............	1⅛	1⅜	1¾	2	2½	3	3½	4	4½
g_2..............	...	...	...	...	...	2	2¼	2½	3
g_3..............	...	...	...	...	...	1¾	2½	3	3
Maximum rivet...	⅝″	¾″	⅞″	⅞″	⅞″	⅞″	⅞″	1″	1⅛″

Regardless of the magnitude of the stress, it is customary to use at least two rivets at the ends of each member. The maximum spacing on centers, called the pitch of rivets, is usually 6″, and occasionally this requirement in conjunction with the length of the gusset plate may determine the number of rivets. The minimum pitch of rivets is 3 diameters of a rivet. It is essential that the two angles which make up a truss member act together, and to accomplish this *stitch rivets* are placed between the joints. For compression members they should not be placed more than 2′ on centers and for tension members not more than 3′.

When a member is continuous at a joint, as for instance *BL* and *CM* at the joint *BCML* (Fig. 82) the rivets are not required to transmit the entire chord stress on either side of the joint. The load transmitted by the rivets is the difference in magnitude of the two loads, in this instance 81,900 − 76,050 or 5,850#, and this is the load used in determining the number of rivets required.

In theory the lines of action of truss members meet in a point at the joints and these lines should coincide with the center of gravity axes of the members. To avoid eccentricity, the lines of rivets should be placed on these lines but, owing to the practical difficulty in driving rivets close to the outstanding legs, the rivets are placed on the gage lines. Standard gage dimensions are given in Table 45. In practice, the lines of action are made to coincide with the gage lines of the sections.

EXAMPLES

Example. Compute the number of rivets in the members *BL* and *LK* about the joint *ABLK*, using 3/8" gusset plates and 3/4" rivets. The allowable unit stresses are to be the stresses used in computing working values in Table 25-A.

Solution. The load on member *BL* is 81,900#, the angles are 7/16" in thickness, the gusset is 3/8", and the rivets are 3/4" in diameter. By referring to Table 25-A we find the following allowable working values:

3/4" rivet, double shear = 11,930#

3/8" gusset, double bearing = 8,440#

For 7/16" angles, no value is given in the table since it is greater than the shearing value; 8,440# is therefore the controlling value, and the number of rivets required in the member *BL* is $\frac{81,900}{8,440}$ or 9+. Use 10 rivets.

In the same manner 8,440# is the controlling value of 1 rivet for the member *LK* and the number required is $\frac{71,010}{8,440}$ or 8+; use 9 rivets.

The number of rivets could be reduced by using a 1/2" gusset plate, in which case the controlling value would be 11,250# (Table 25-A), and 8 rivets would be required in *BL* and 7 rivets in *LK*.

Example. Compute the number of rivets required in the members about the joint *BCML*.

Solution. The load on member *BL* is 81,900#, and that on *CM* is 76,050#. Since the two 4" x 3½" x 7/16" angles are continuous in the upper chord, the load to be transferred by the rivets

at this joint is 81,900 − 76,050 or 5,850#. The controlling value of 1 rivet, as explained in the preceding example, is 8,440#. Actually only 1 rivet is required but 3 will be used to avoid eccentricity, the rivets being placed on the gage lines.

The member *LM* has a stress of 10,170#, the section is composed of two 2½″ x 2″ x 5/16″ angles, and the gusset is 3/8″. Referring to Table 25-A we find the allowable working values to be

¾″ rivet, double shear	= 11,930#
3/8″ gusset, double bearing	= 8,440#
Two 5/16″ angles, single bearing, 2 × 5,630	= 11,260#

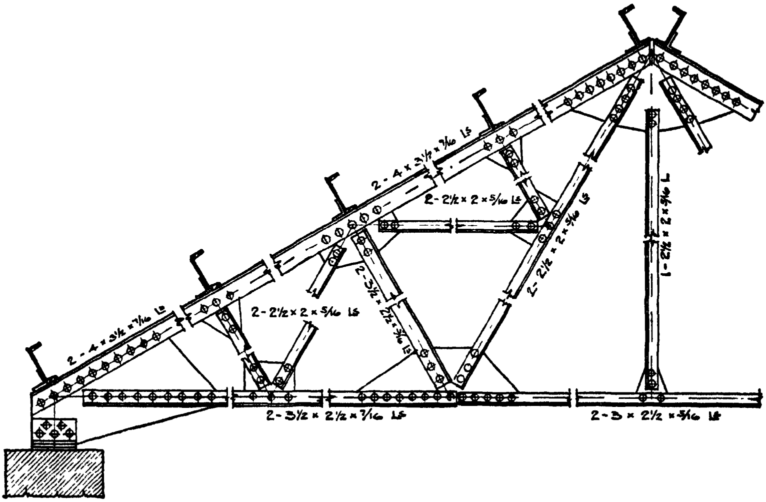


Fig. 83

The controlling value is 8,440#, and the number of rivets required is $\frac{10,170}{8,440} = 1+$. Use 2 rivets.

115. **Shop Drawings.** The drawings used to fabricate steel work are known as shop drawings. These drawings give complete details, spacing of rivets, and dimensions of members, and gusset plates. Shop drawings are made by the fabricating company from the general information given by the architect (see Fig. 83) and are submitted to the architect for approval before the manufacturing process is begun.

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